

Basic Engineering for Building Plans Examiners & Inspectors

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Basic Forces & Stresses



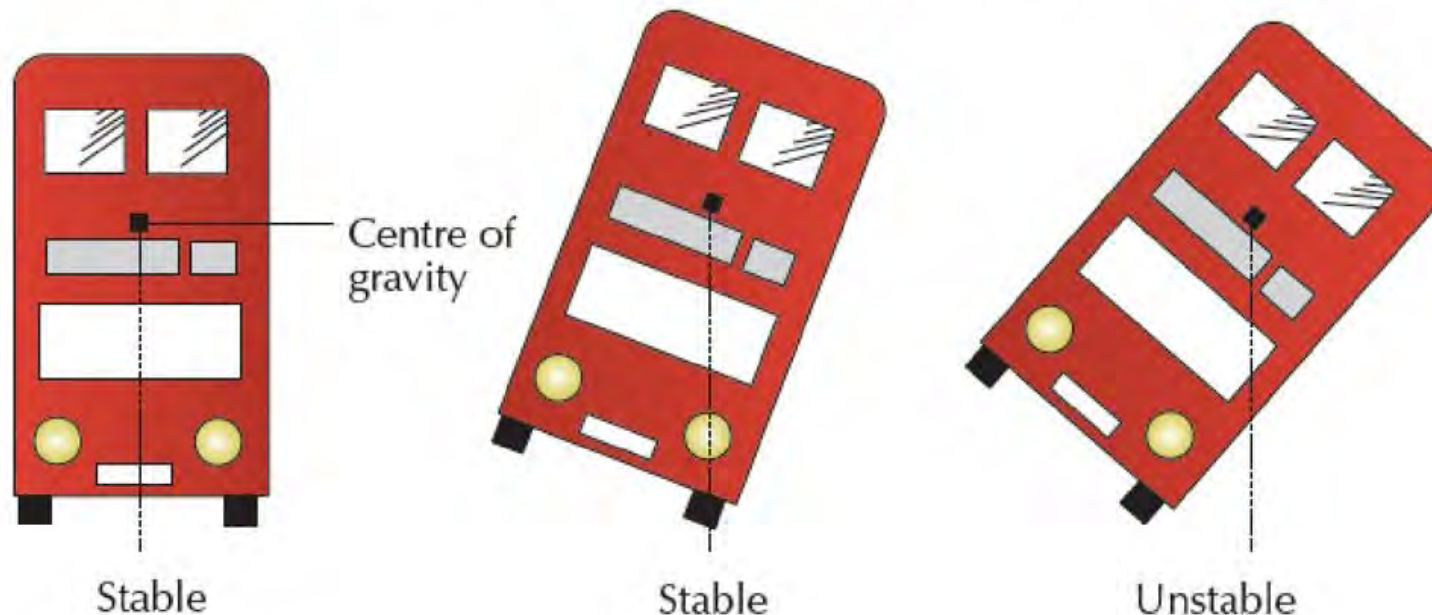
Center of gravity

- ▶ A point from which the weight of a body or system may be considered to act.
- ▶ In uniform gravity it is the same as the center of mass.



Stability

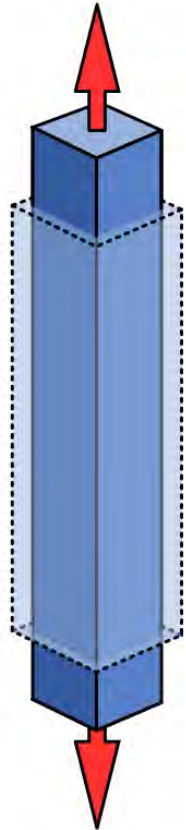
- An object will topple over if its centre of gravity passes outside its base.



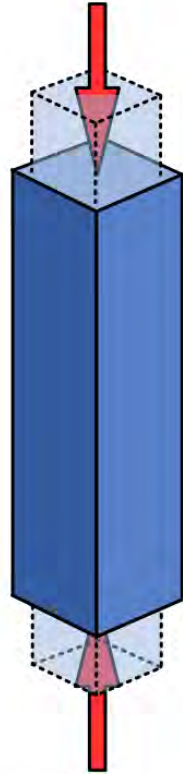
Force

Force is any action that tends to maintain or alter the motion of a body or to distort it.

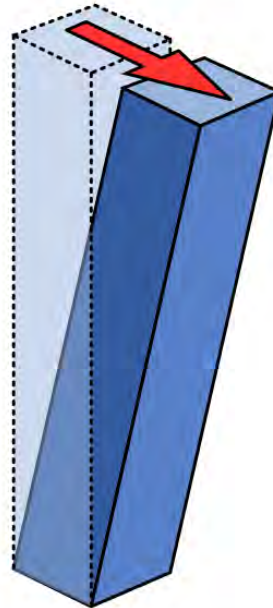




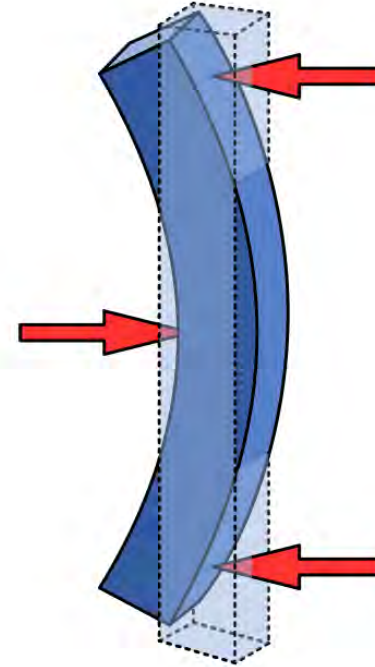
tension



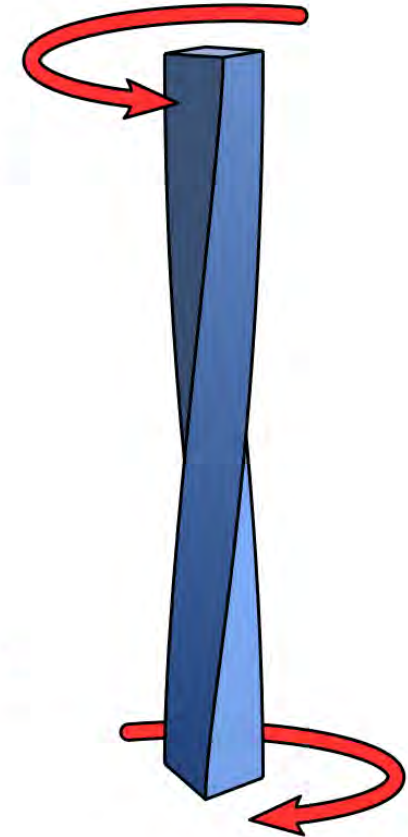
com-
pression



shear



bending



torsion

STRESS

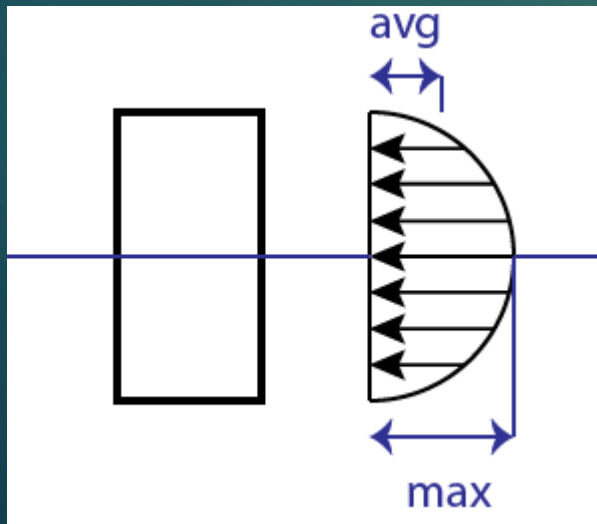
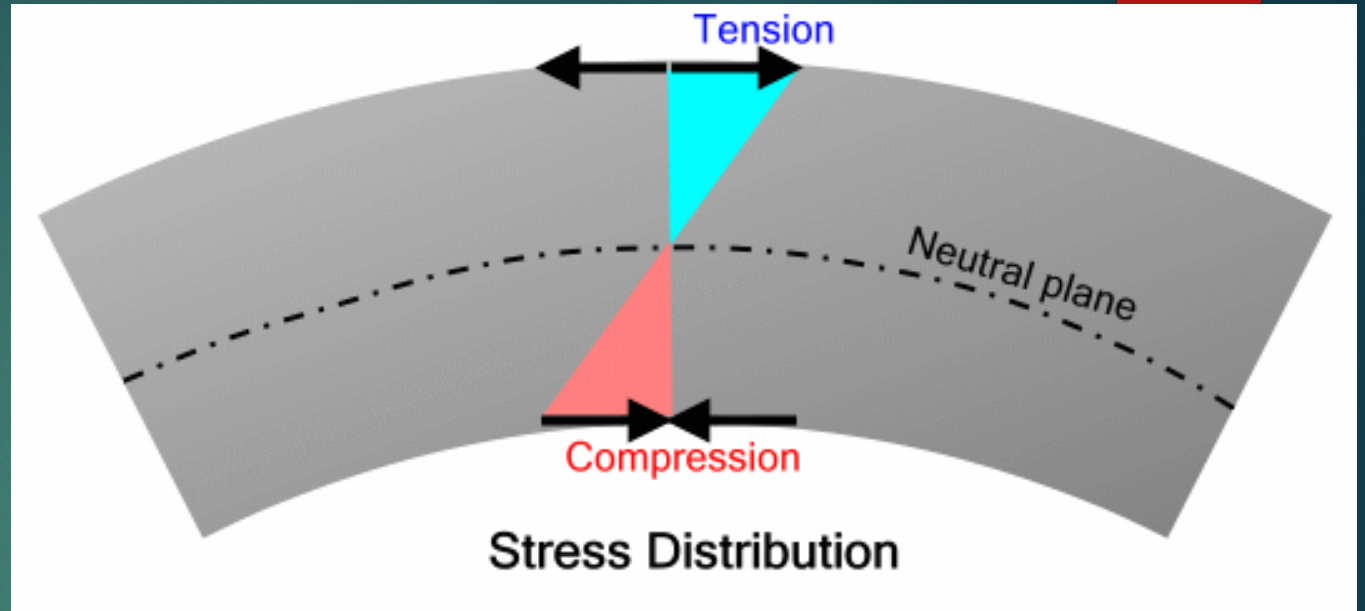
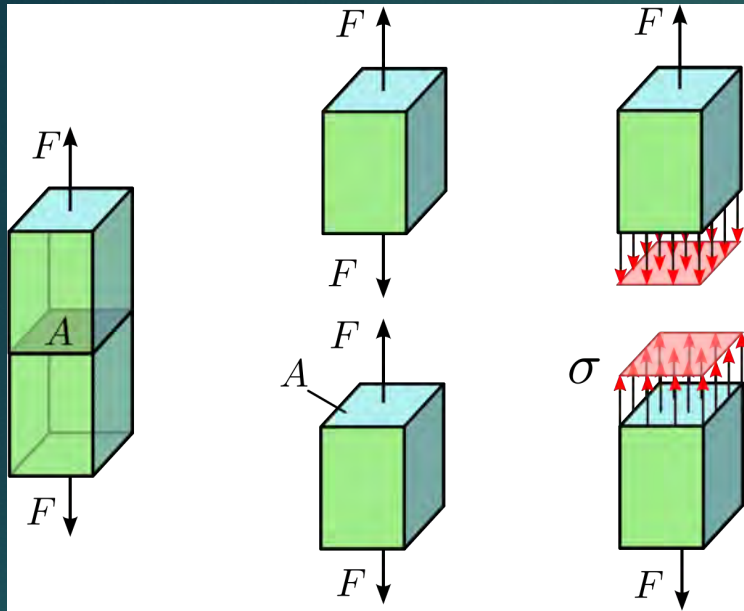
- ▶ The term stress is used to express the loading in terms of force applied to a certain cross-sectional area of an object. From the perspective of loading, stress is the applied force or system of forces that tends to deform a body.
- ▶ Stress is the internal distribution of forces within a body that balance and react to the loads applied to it. The stress distribution may or may not be uniform, depending on the nature of the loading condition.
- ▶ Expressed in units of force per area.

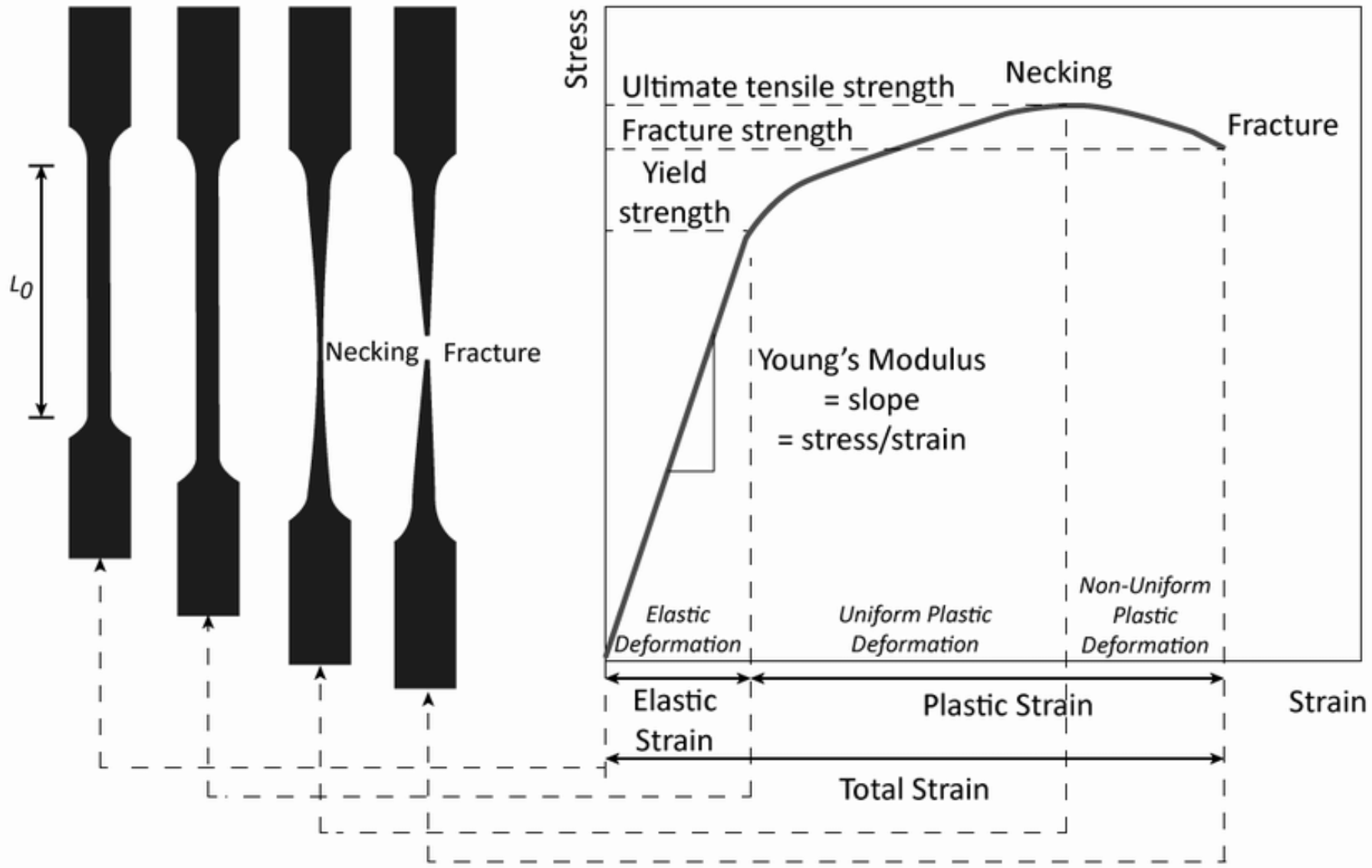


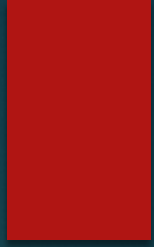
Strain

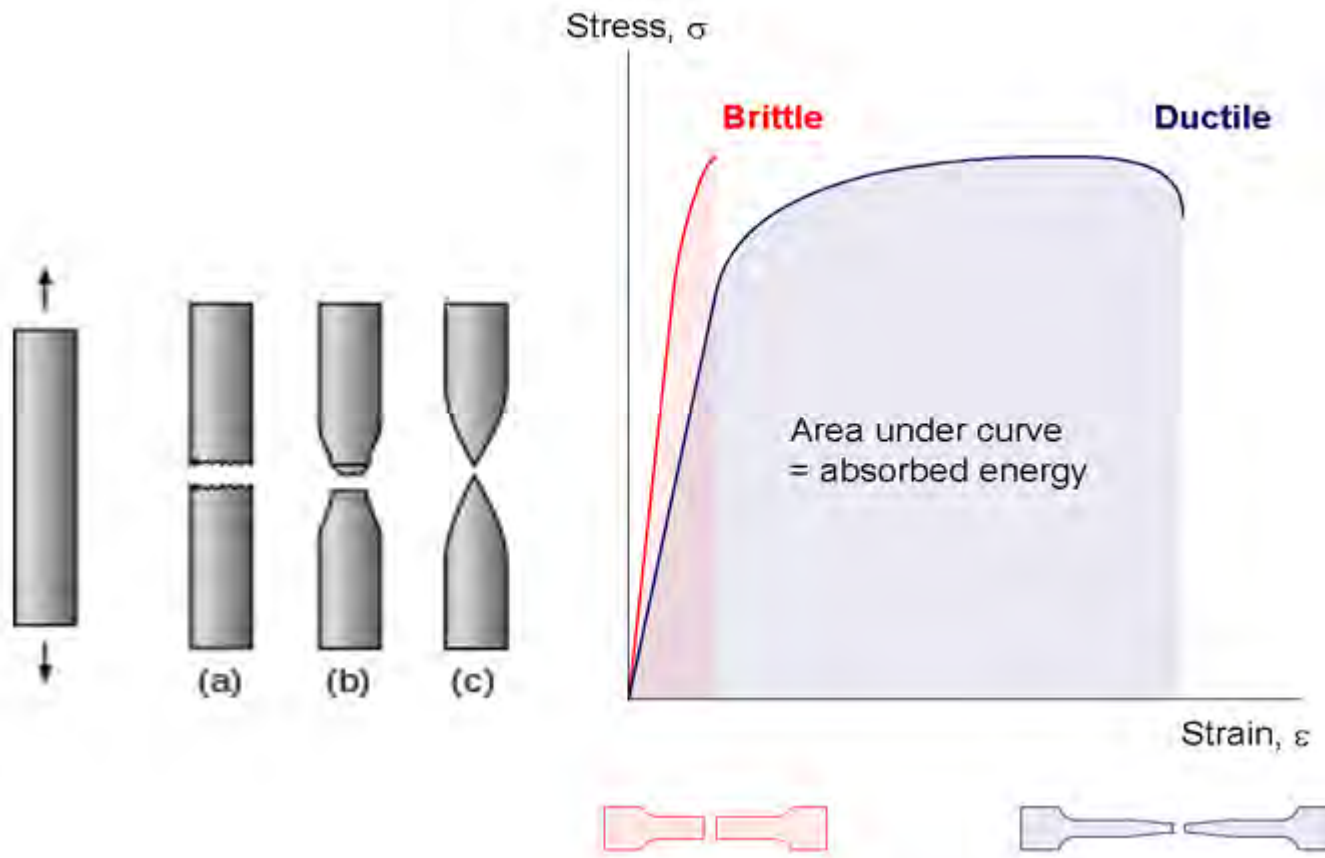
- ▶ Strain is the response of a system to an applied stress
- ▶ Engineering strain is defined as the amount of deformation in the direction of the applied force divided by the initial length of the material. This results in a unitless number, although it is often left in the unsimplified form, such as inches per inch or meters per meter.
- ▶ the strain distribution may or may not be uniform in a complex structural element, depending on the nature of the loading condition.



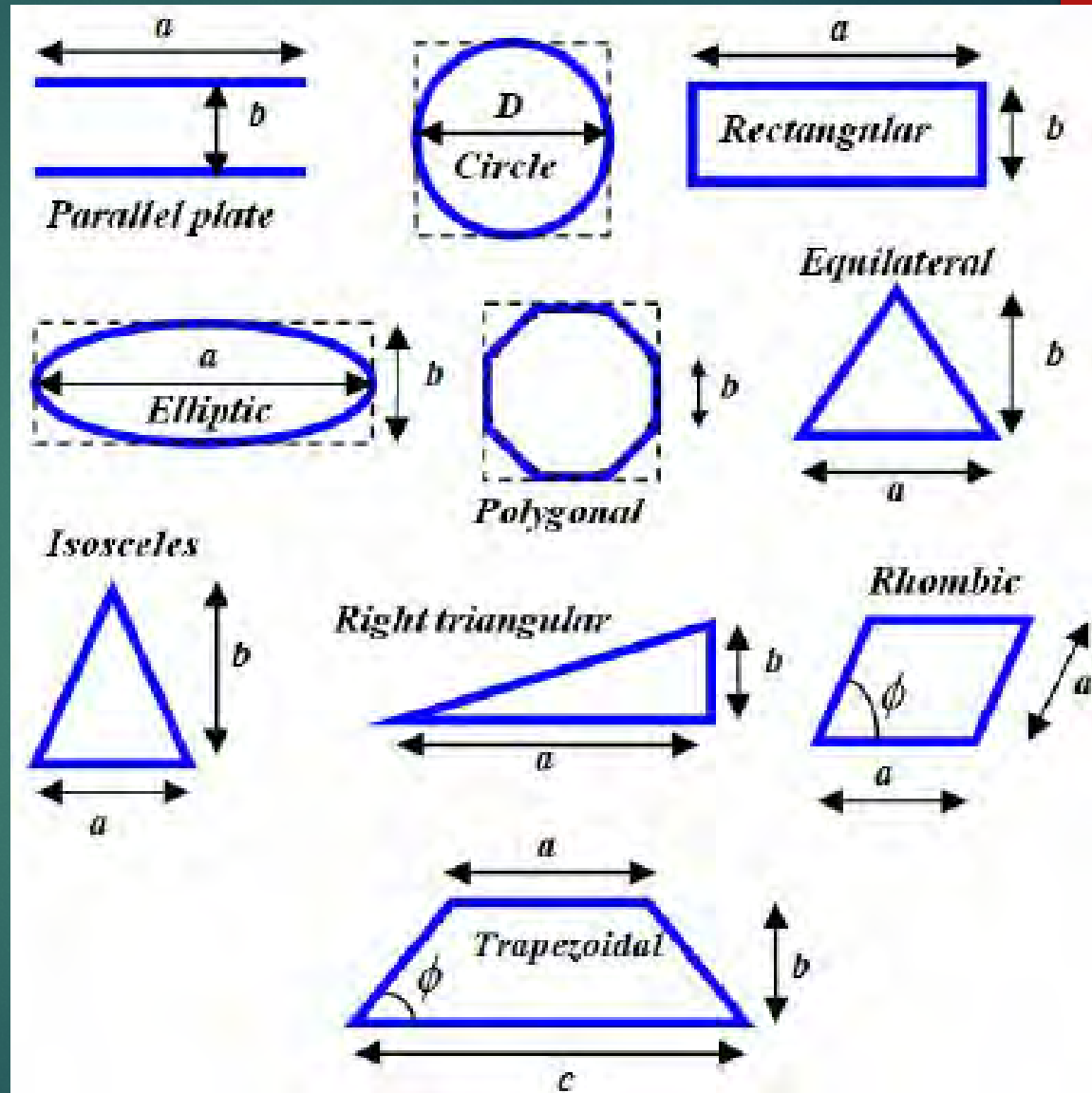




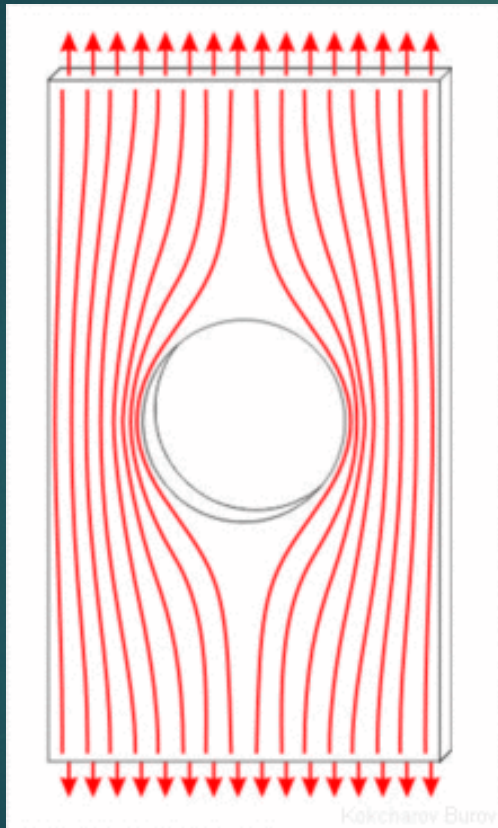




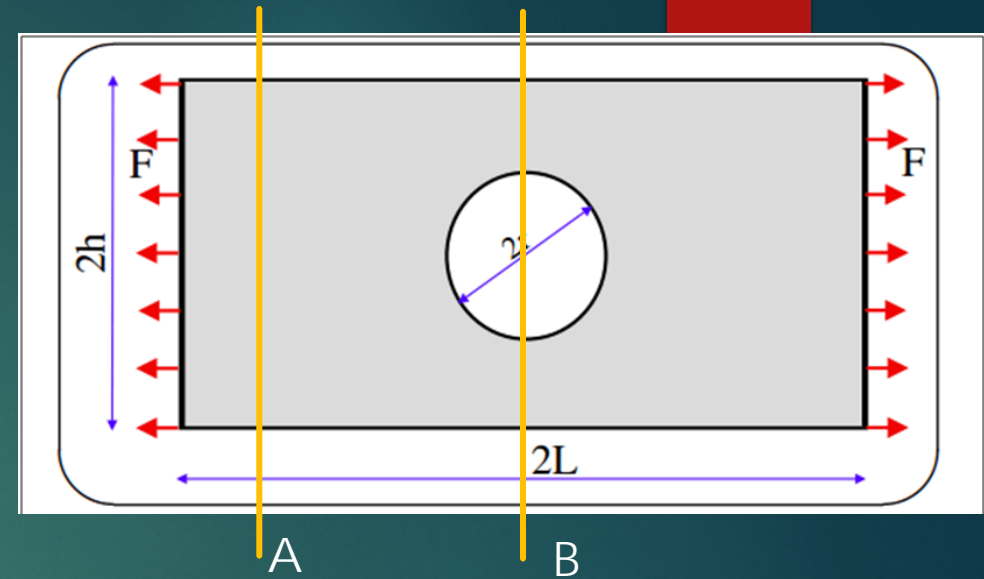
Area



Gross Area vs. Net area



Kakchany Buray



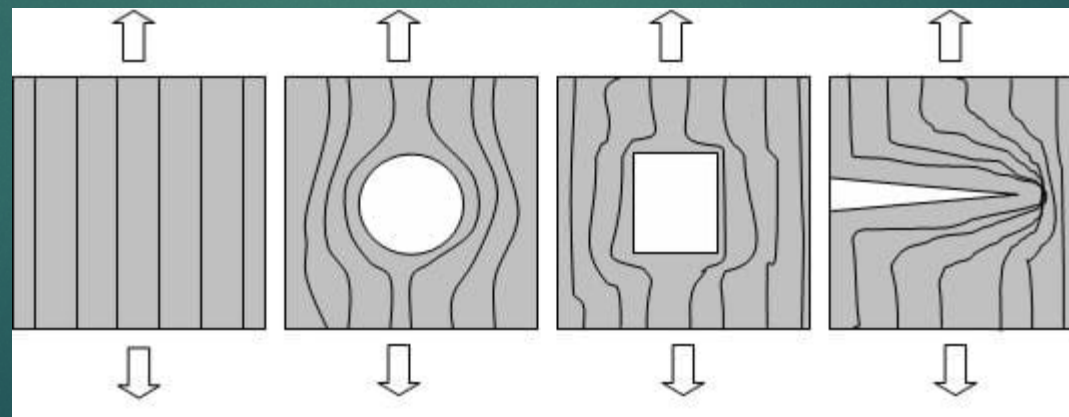
A

Area = 0.625

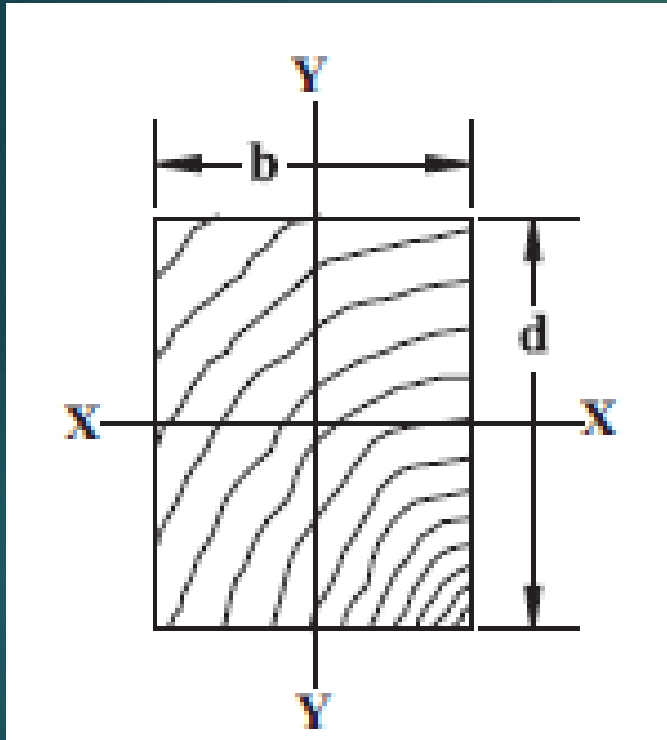


B

Area = 0.5



Moment of Inertia



$$I_{xx} = \frac{bd^3}{12}$$

$$I_{yy} = \frac{db^3}{12}$$

Section Modulus

$$S_{xx} = \frac{I_{xx}}{d/2} = \frac{bd^3}{12} \frac{2}{d} = \frac{bd^2}{6}$$

$$S_{yy} = \frac{I_{yy}}{b/2} = \frac{db^3}{12} \frac{2}{b} = \frac{db^2}{6}$$



Cross-Sectional Properties

| Nominal Size b x d | Standard Dressed Size (S4S) b x d in. x in. | Area of Section A in. ² | X-X AXIS | | Y-Y AXIS | |
|---|---|--|--|--|--|--|
| | | | Section Modulus S _{xx} in. ³ | Moment of Inertia I _{xx} in. ⁴ | Section Modulus S _{yy} in. ³ | Moment of Inertia I _{yy} in. ⁴ |
| Boards¹ | | | | | | |
| 1 x 3 | 3/4 x 2-1/2 | 1.875 | 0.781 | 0.977 | 0.234 | 0.088 |
| 1 x 4 | 3/4 x 3-1/2 | 2.625 | 1.531 | 2.680 | 0.328 | 0.123 |
| 1 x 6 | 3/4 x 5-1/2 | 4.125 | 3.781 | 10.40 | 0.516 | 0.193 |
| 1 x 8 | 3/4 x 7-1/4 | 5.438 | 6.570 | 23.82 | 0.680 | 0.255 |
| 1 x 10 | 3/4 x 9-1/4 | 6.938 | 10.70 | 49.47 | 0.867 | 0.325 |
| 1 x 12 | 3/4 x 11-1/4 | 8.438 | 15.82 | 88.99 | 1.055 | 0.396 |
| Dimension Lumber (see NDS 4.1.3.2) and Decking (see NDS 4.1.3.5) | | | | | | |
| 2 x 3 | 1-1/2 x 2-1/2 | 3.750 | 1.56 | 1.953 | 0.938 | 0.703 |
| 2 x 4 | 1-1/2 x 3-1/2 | 5.250 | 3.06 | 5.359 | 1.313 | 0.984 |
| 2 x 5 | 1-1/2 x 4-1/2 | 6.750 | 5.06 | 11.39 | 1.688 | 1.266 |
| 2 x 6 | 1-1/2 x 5-1/2 | 8.250 | 7.56 | 20.80 | 2.063 | 1.547 |
| 2 x 8 | 1-1/2 x 7-1/4 | 10.88 | 13.14 | 47.63 | 2.719 | 2.039 |
| 2 x 10 | 1-1/2 x 9-1/4 | 13.88 | 21.39 | 98.93 | 3.469 | 2.602 |
| 2 x 12 | 1-1/2 x 11-1/4 | 16.88 | 31.64 | 178.0 | 4.219 | 3.164 |
| 2 x 14 | 1-1/2 x 13-1/4 | 19.88 | 43.89 | 290.8 | 4.969 | 3.727 |
| 3 x 4 | 2-1/2 x 3-1/2 | 8.75 | 5.10 | 8.932 | 3.646 | 4.557 |
| 3 x 5 | 2-1/2 x 4-1/2 | 11.25 | 8.44 | 18.98 | 4.688 | 5.859 |
| 3 x 6 | 2-1/2 x 5-1/2 | 13.75 | 12.60 | 34.66 | 5.729 | 7.161 |
| 3 x 8 | 2-1/2 x 7-1/4 | 18.13 | 21.90 | 79.39 | 7.552 | 9.440 |
| 3 x 10 | 2-1/2 x 9-1/4 | 23.13 | 35.65 | 164.9 | 9.635 | 12.04 |
| 3 x 12 | 2-1/2 x 11-1/4 | 28.13 | 52.73 | 296.6 | 11.72 | 14.65 |
| 3 x 14 | 2-1/2 x 13-1/4 | 33.13 | 73.15 | 484.6 | 13.80 | 17.25 |
| 3 x 16 | 2-1/2 x 15-1/4 | 38.13 | 96.90 | 738.9 | 15.89 | 19.86 |
| 4 x 4 | 3-1/2 x 3-1/2 | 12.25 | 7.15 | 12.51 | 7.146 | 12.51 |
| 4 x 5 | 3-1/2 x 4-1/2 | 15.75 | 11.81 | 26.58 | 9.188 | 16.08 |
| 4 x 6 | 3-1/2 x 5-1/2 | 19.25 | 17.65 | 48.53 | 11.23 | 19.65 |
| 4 x 8 | 3-1/2 x 7-1/4 | 25.38 | 30.66 | 111.1 | 14.80 | 25.90 |
| 4 x 10 | 3-1/2 x 9-1/4 | 32.38 | 49.91 | 230.8 | 18.89 | 33.05 |
| 4 x 12 | 3-1/2 x 11-1/4 | 39.38 | 73.83 | 415.3 | 22.97 | 40.20 |
| 4 x 14 | 3-1/2 x 13-1/4 | 46.38 | 102.41 | 678.5 | 27.05 | 47.34 |
| 4 x 16 | 3-1/2 x 15-1/4 | 53.38 | 135.66 | 1034 | 31.14 | 54.49 |



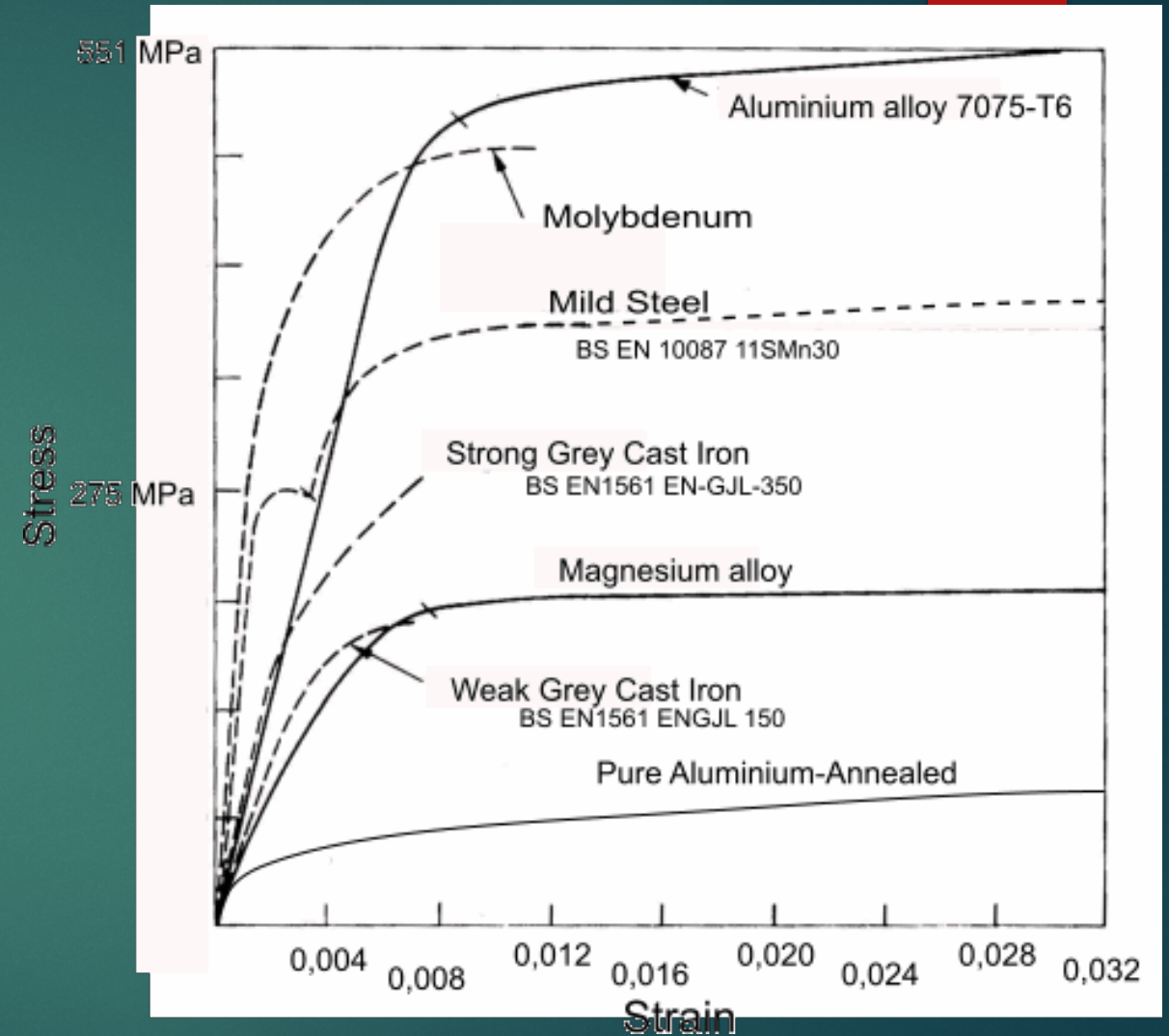
Cross-Sectional Properties

| Nominal Size b x d | Standard Dressed Size (S4S) b x d in. x in. | Area of Section A in. ² | X-X AXIS | | Y-Y AXIS | |
|--|---|--|--|--|--|--|
| | | | Section Modulus S _{xx} in. ³ | Moment of Inertia I _{xx} in. ⁴ | Section Modulus S _{yy} in. ³ | Moment of Inertia I _{yy} in. ⁴ |
| Timbers (5" x 5" and larger)² | | | | | | |
| Post and Timber (see NDS 4.1.3.4 and NDS 4.1.5.3) | | | | | | |
| 5 x 5 | 4-1/2 x 4-1/2 | 20.25 | 15.19 | 34.17 | 15.19 | 34.17 |
| 6 x 6 | 5-1/2 x 5-1/2 | 30.25 | 27.73 | 76.26 | 27.73 | 76.26 |
| 6 x 8 | 5-1/2 x 7-1/2 | 41.25 | 51.56 | 193.4 | 37.81 | 104.0 |
| 8 x 8 | 7-1/2 x 7-1/2 | 56.25 | 70.31 | 263.7 | 70.31 | 263.7 |
| 8 x 10 | 7-1/2 x 9-1/2 | 71.25 | 112.8 | 535.9 | 89.06 | 334.0 |
| 10 x 10 | 9-1/2 x 9-1/2 | 90.25 | 142.9 | 678.8 | 142.9 | 678.8 |
| 10 x 12 | 9-1/2 x 11-1/2 | 109.3 | 209.4 | 1204 | 173.0 | 821.7 |
| 12 x 12 | 11-1/2 x 11-1/2 | 132.3 | 253.5 | 1458 | 253.5 | 1458 |
| 12 x 14 | 11-1/2 x 13-1/2 | 155.3 | 349.3 | 2358 | 297.6 | 1711 |
| 14 x 14 | 13-1/2 x 13-1/2 | 182.3 | 410.1 | 2768 | 410.1 | 2768 |
| 14 x 16 | 13-1/2 x 15-1/2 | 209.3 | 540.6 | 4189 | 470.8 | 3178 |
| 16 x 16 | 15-1/2 x 15-1/2 | 240.3 | 620.6 | 4810 | 620.6 | 4810 |
| 16 x 18 | 15-1/2 x 17-1/2 | 271.3 | 791.1 | 6923 | 700.7 | 5431 |
| 18 x 18 | 17-1/2 x 17-1/2 | 306.3 | 893.2 | 7816 | 893.2 | 7816 |
| 18 x 20 | 17-1/2 x 19-1/2 | 341.3 | 1109 | 10813 | 995.3 | 8709 |
| 20 x 20 | 19-1/2 x 19-1/2 | 380.3 | 1236 | 12049 | 1236 | 12049 |
| 20 x 22 | 19-1/2 x 21-1/2 | 419.3 | 1502 | 16150 | 1363 | 13285 |
| 22 x 22 | 21-1/2 x 21-1/2 | 462.3 | 1656 | 17806 | 1656 | 17806 |
| 22 x 24 | 21-1/2 x 23-1/2 | 505.3 | 1979 | 23252 | 1810 | 19463 |
| 24 x 24 | 23-1/2 x 23-1/2 | 552.3 | 2163 | 25415 | 2163 | 25415 |



Material stiffness

- ▶ Steel – 29000 ksi
- ▶ Concrete – 3605 ksi ($f'c=4000$ psi)
- ▶ Wood – 1600 ksi (Varies by sp/gr)
- ▶ 6061 Aluminum – 10000 ksi



Material strength



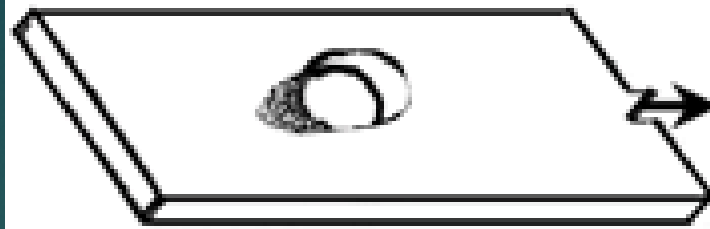


Tension Failure

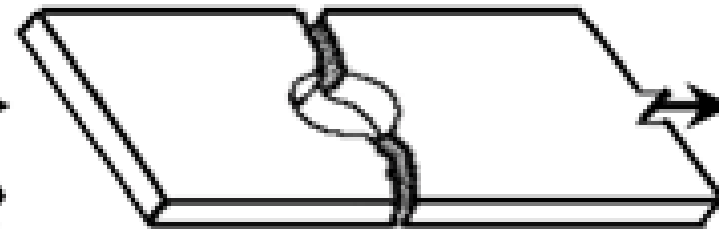
Failure based on

- Material strength properties
- Cross-sectional area
- Net Area
- relatively independent of shape

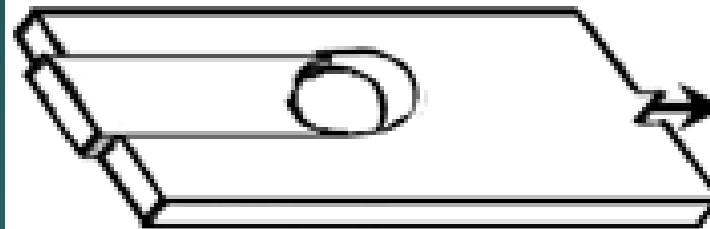




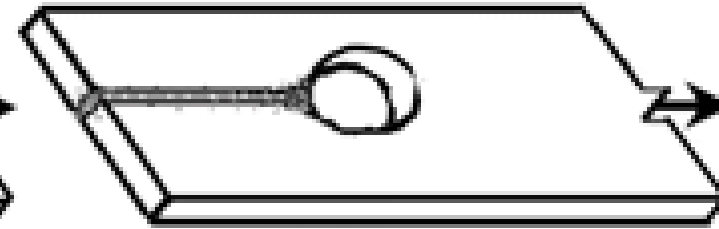
(a) Bearing failure



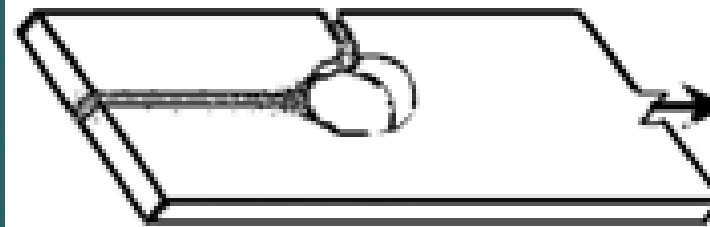
(b) Net-tension failure



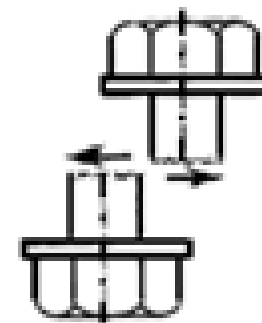
(c) Shear-out failure



(d) Cleavage failure



(e) Cleavage-tension failure



(f) Bolt failure

Compression Failure

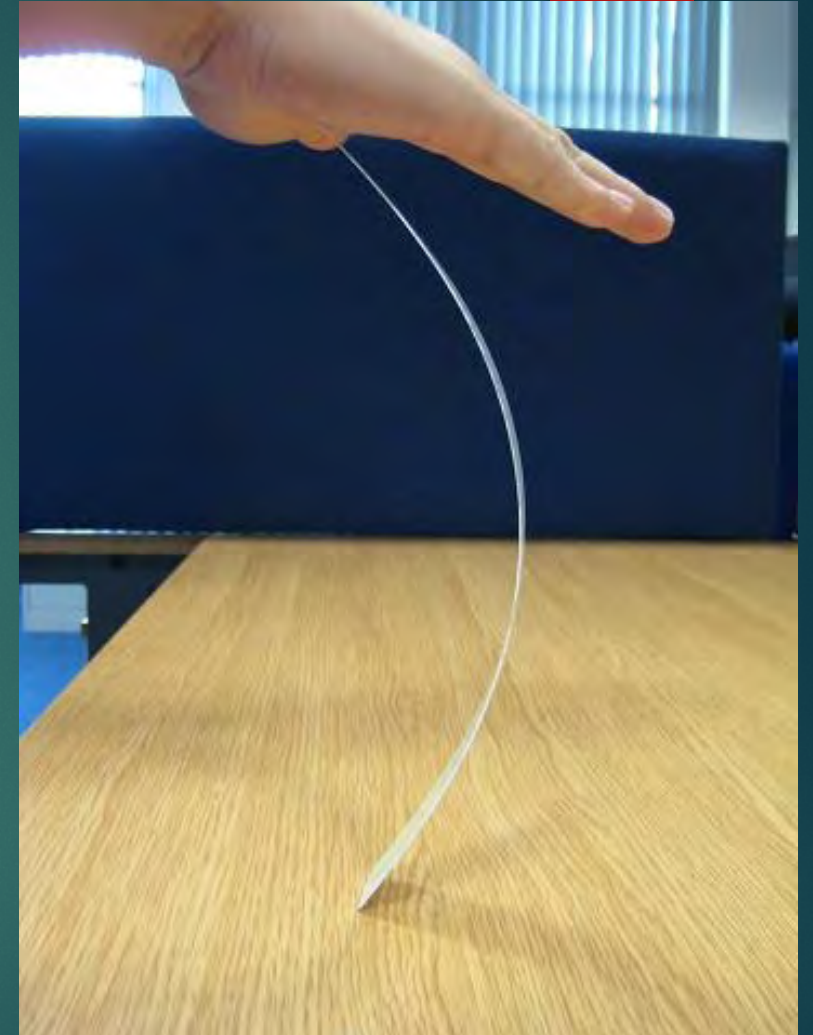
- ▶ Short Members (Columns)
 - ▶ Crushing failure
 - ▶ Failure based on
 - ▶ Material strength properties
 - ▶ Cross-sectional area & relatively independent of shape
 - ▶ Sudden progressive failure type
- ▶ Long Members (Columns)
 - ▶ Buckling failure
 - ▶ Failure based On
 - ▶ Column length
 - ▶ Member stiffness, geometric configuration & material load-deformation relationships
 - ▶ Sudden progressive failure type



Crushing



Buckling

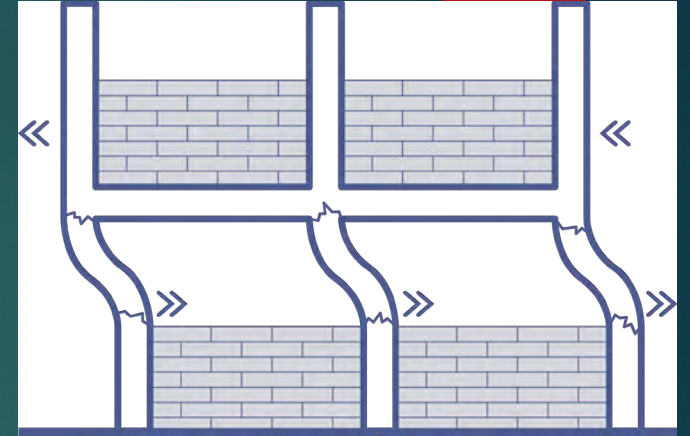


Euler Formula

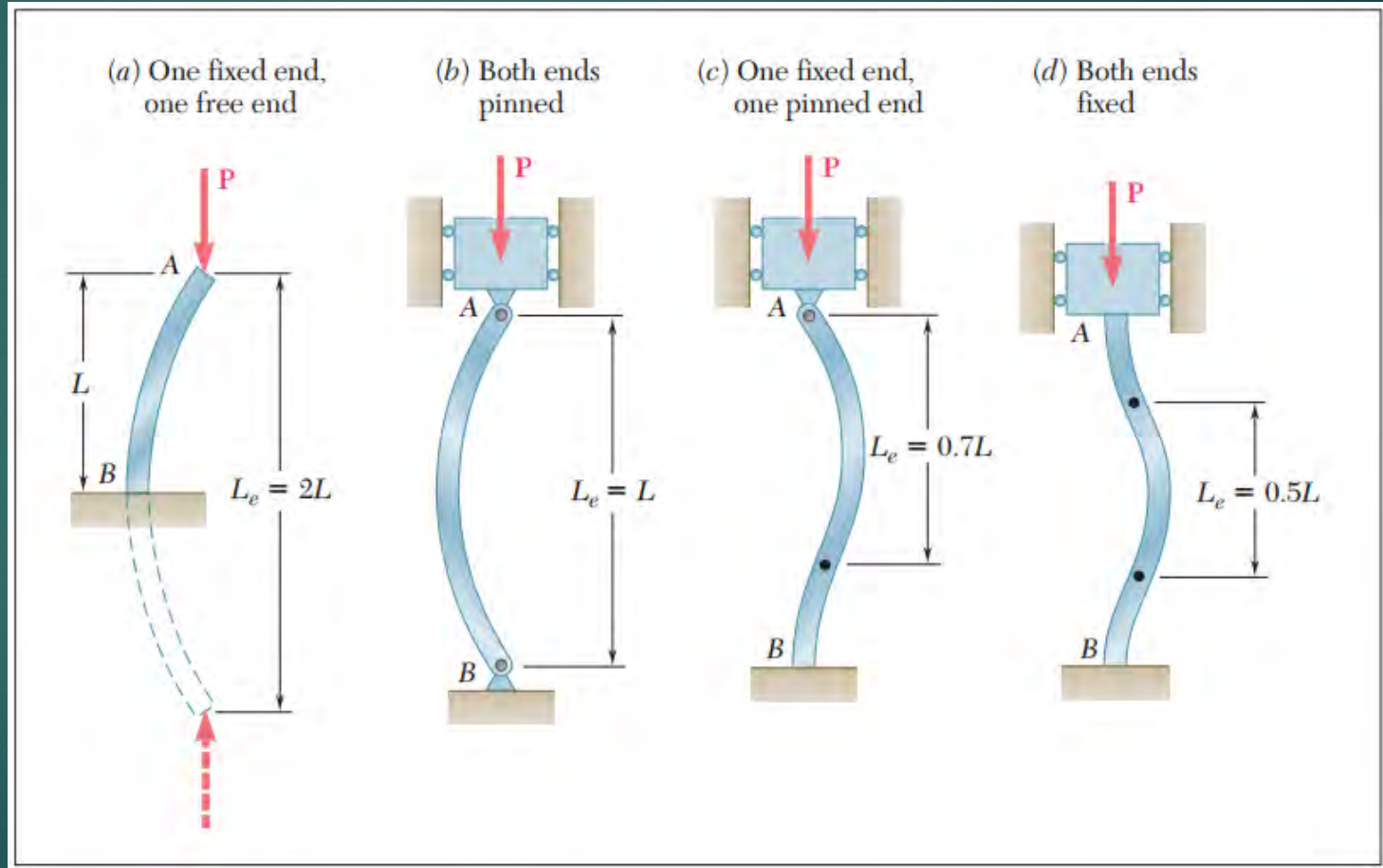
$$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$



Length



End conditions



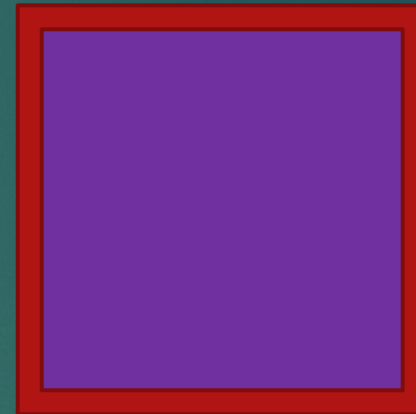
Cross-sectional configuration



1.0 x 1.0
A=1
 $l_x = l_y = 0.083$



2 x 0.5
A=1
 $l_x = 0.33$
 $l_y = 0.021$



2.125 x 2.125 x 0.125
A=1
 $l_x = l_y = 0.67$

Euler Formula

$$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

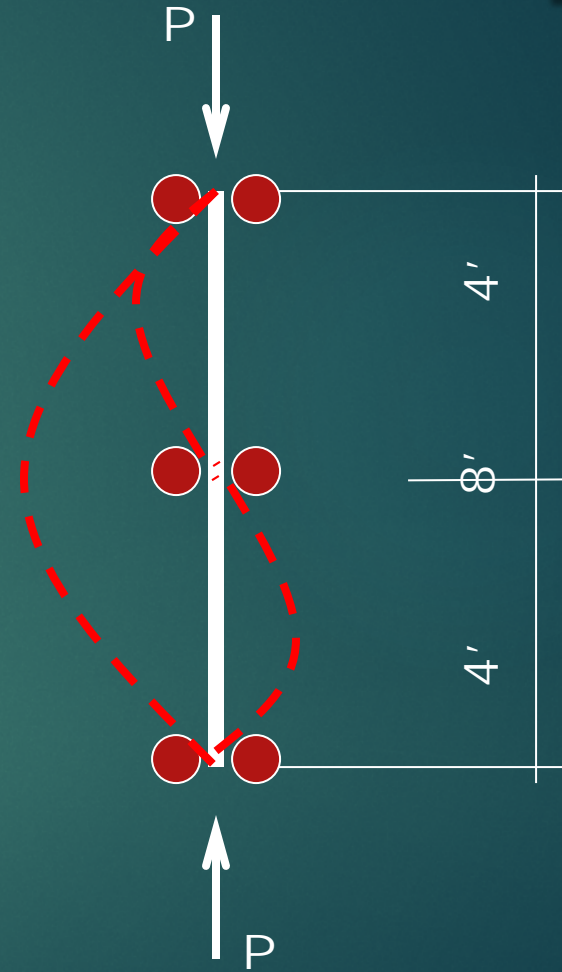


Long Column Capacity

2 x 4

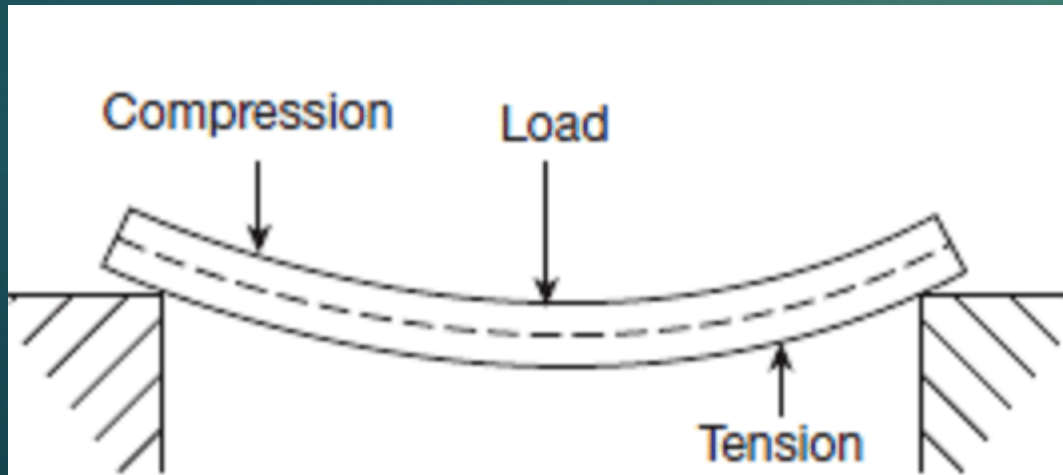
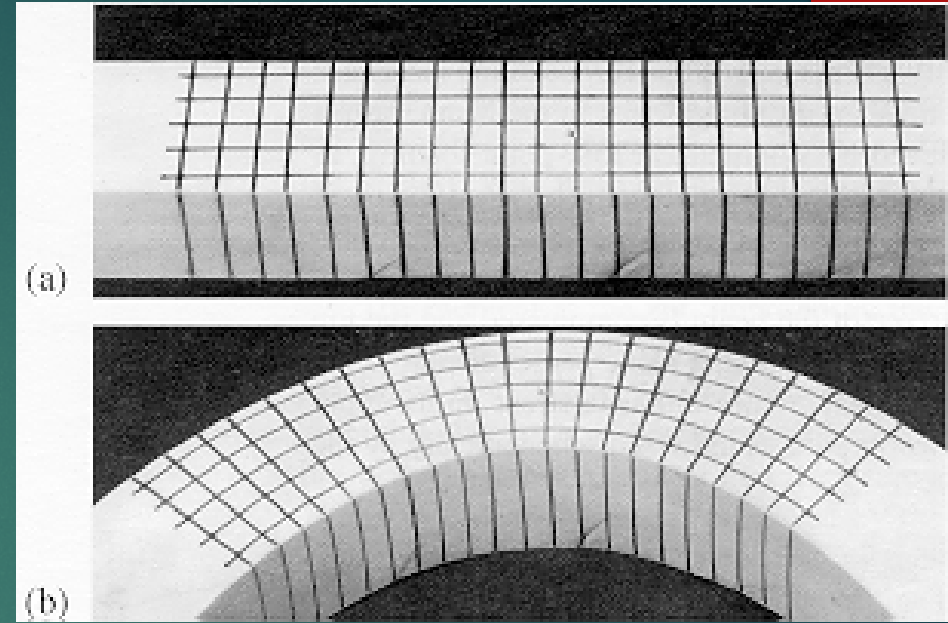
$$L_e = 4' \quad P_{cr} = 11,900 \text{ lbs}$$

$$L_e = 8' \quad P_{cr} = 2,980 \text{ lbs}$$

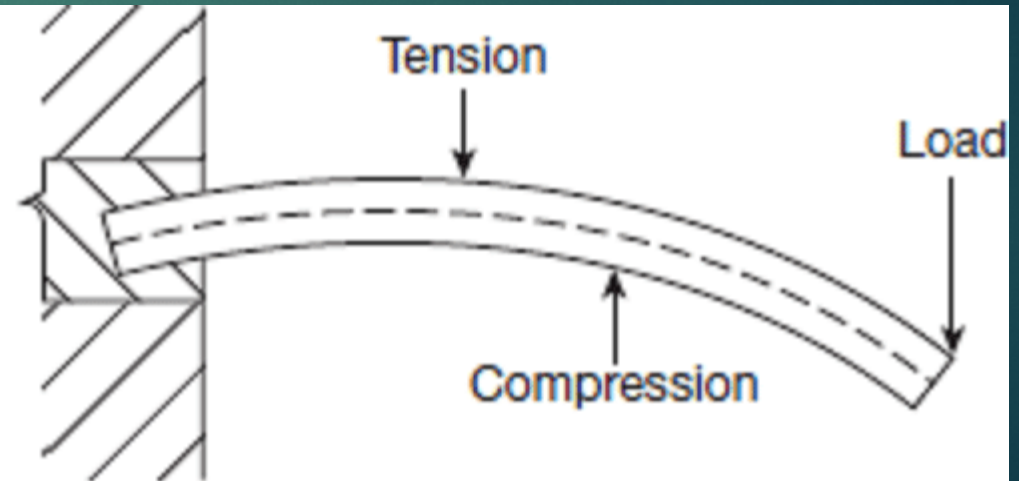


Bending

- ▶ Combination of tension and compression
- ▶ Not efficient
- ▶ Neutral Axis

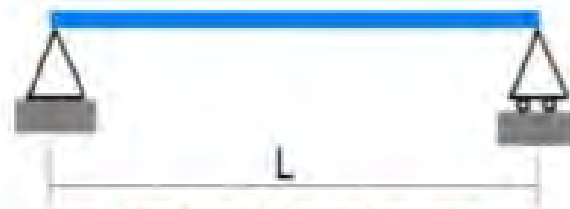


(a) Simply supported beam

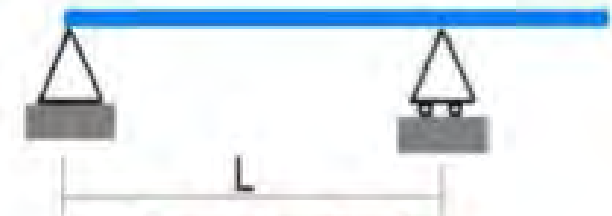


(b) Cantilever beam

Beam types



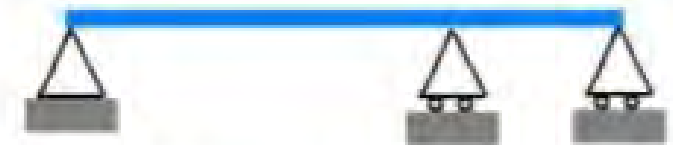
(a) simply supported beams .



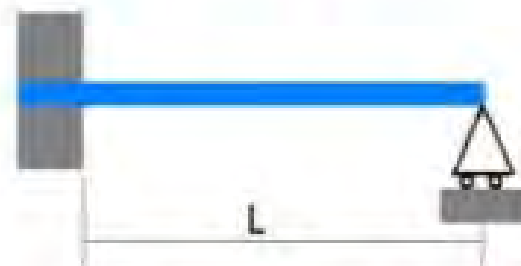
(b) over hanging beam ..



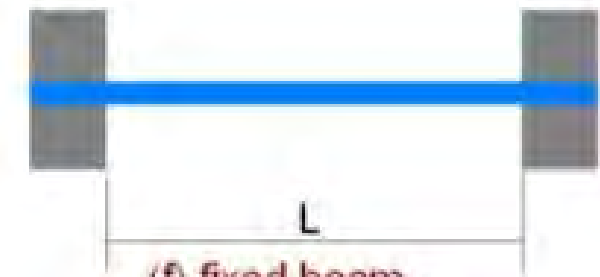
(c) cantiliver beam .



(d) continous beam .



(e) beam fixed at one end and simply supported at the other end .



(f) fixed beam .

Lateral torsional buckling





Redundancy

- ▶ The inclusion of extra components which are not strictly necessary to functioning, in case of failure in other components.
- ▶ Adds cost

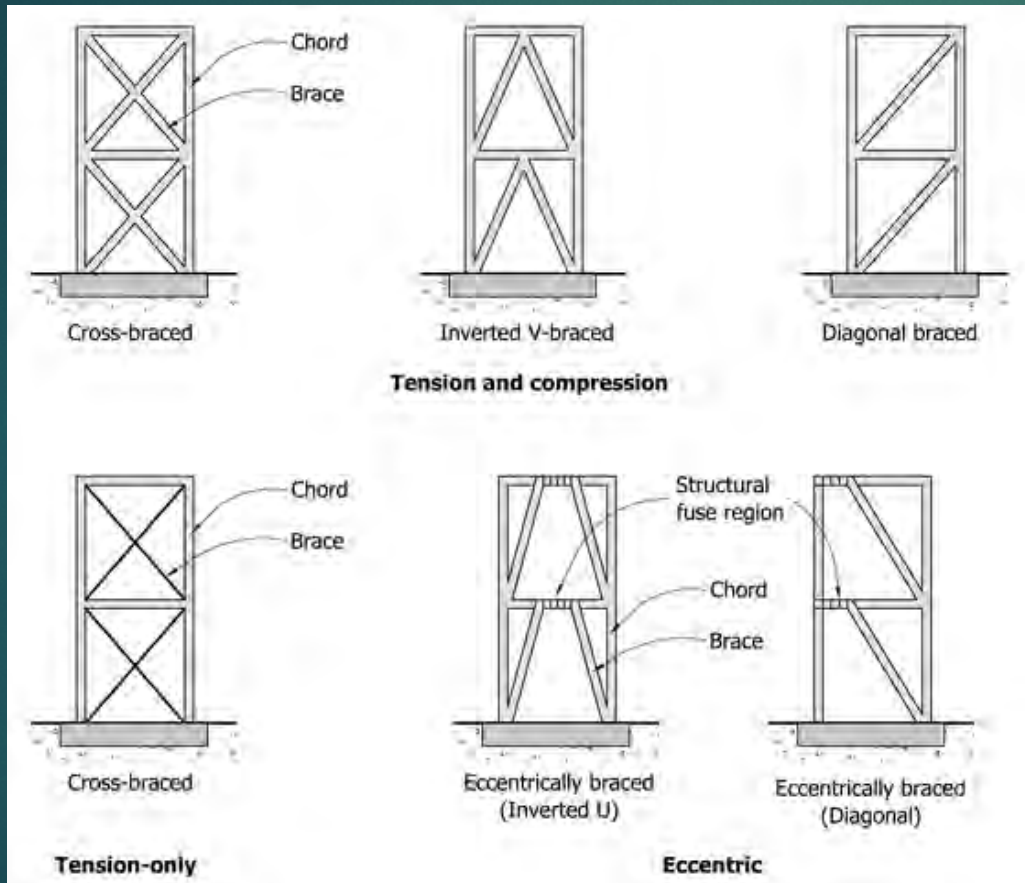


Frame types

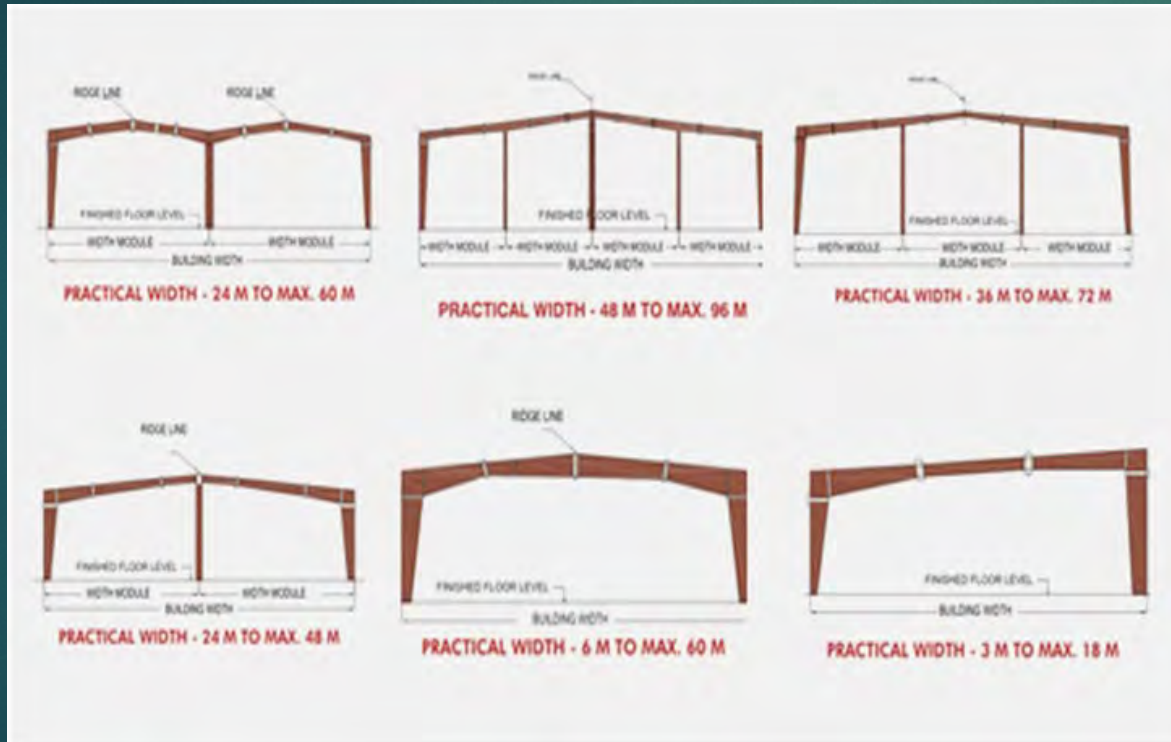
- ▶ Braced
- ▶ Rigid
- ▶ Diaphragms



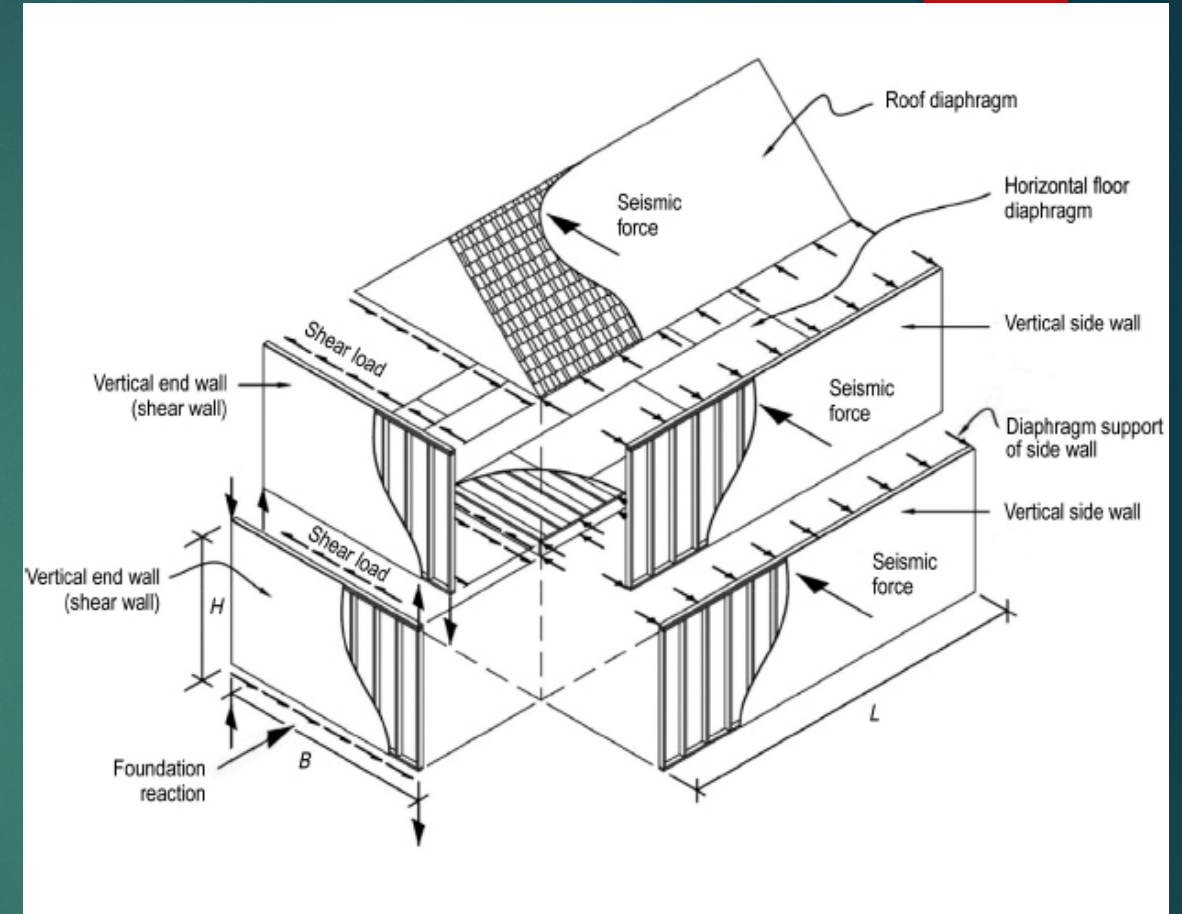
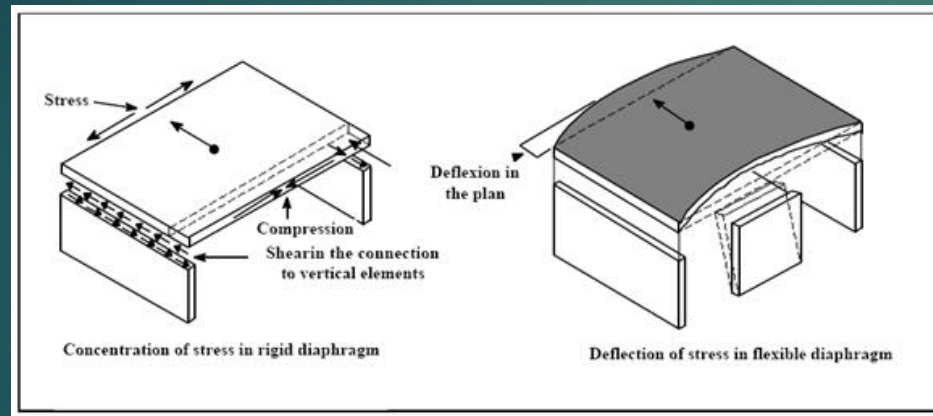
Braced frames



Rigid frames



diaphragms



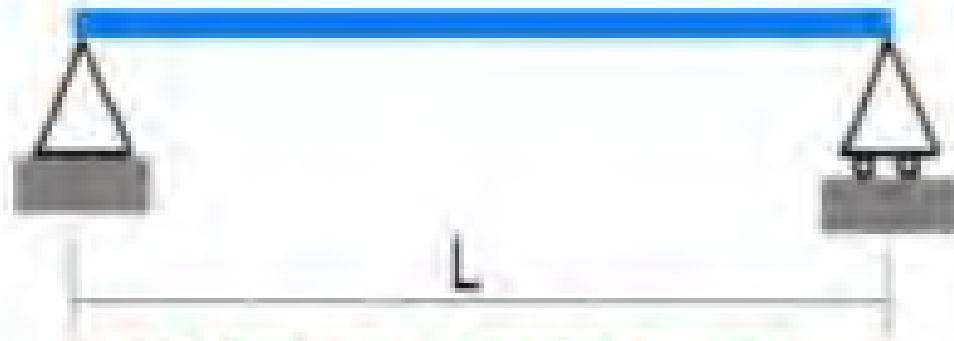
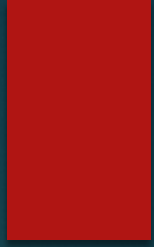
Equilibrium

$$\blacktriangleright \sum F_H = 0$$

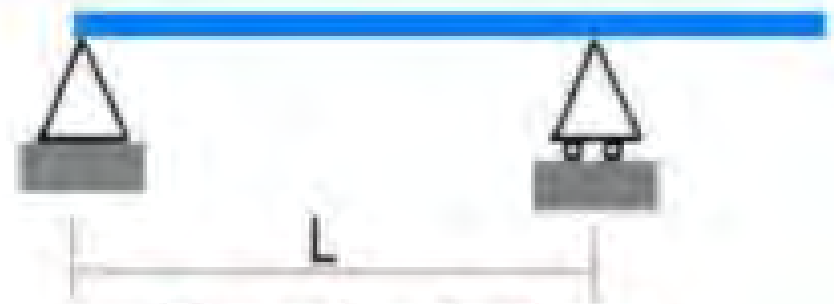
$$\blacktriangleright \sum F_V = 0$$

$$\blacktriangleright \sum M = 0$$





(a) simply supported beams .



(b) over hanging beam .

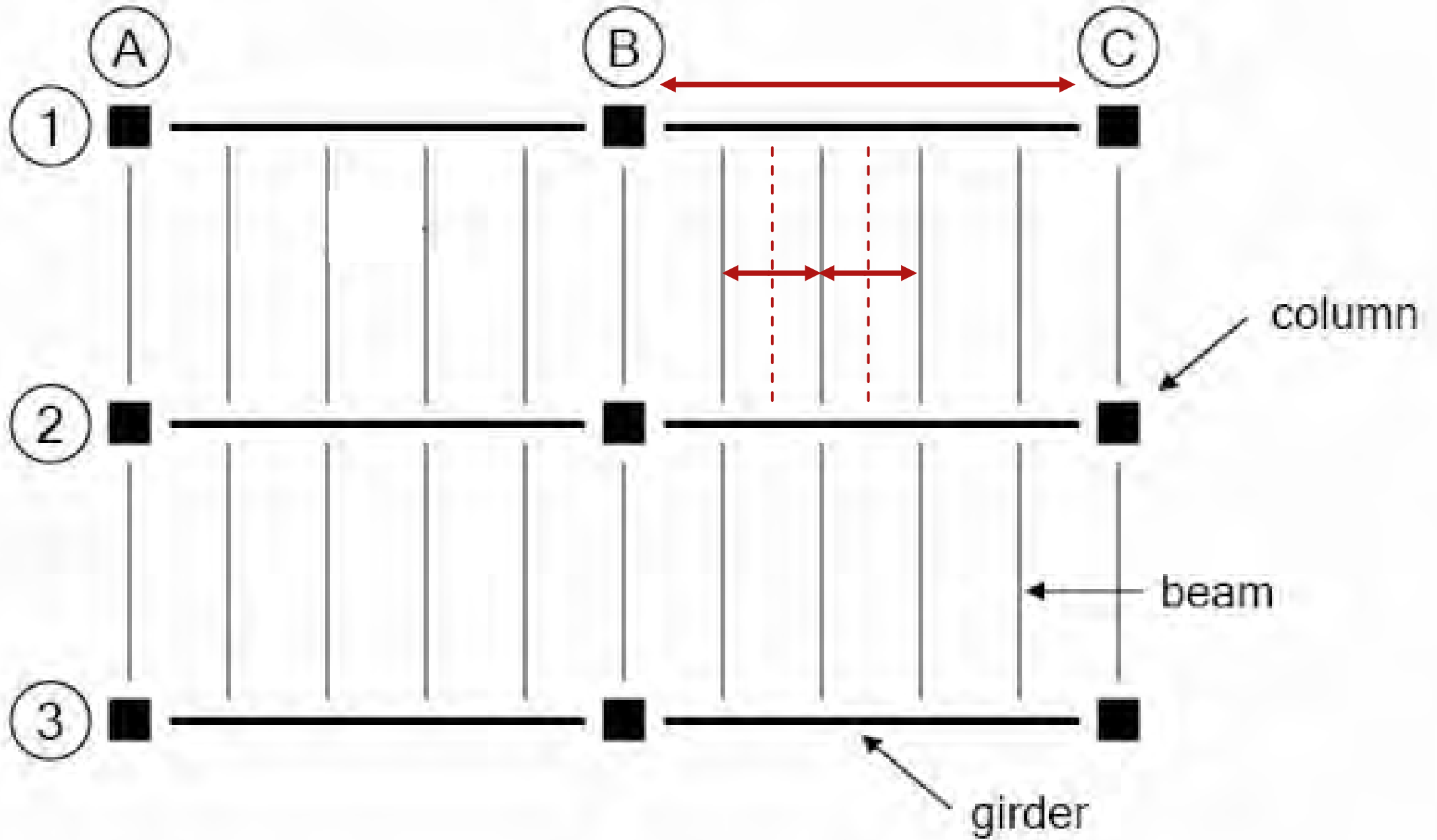


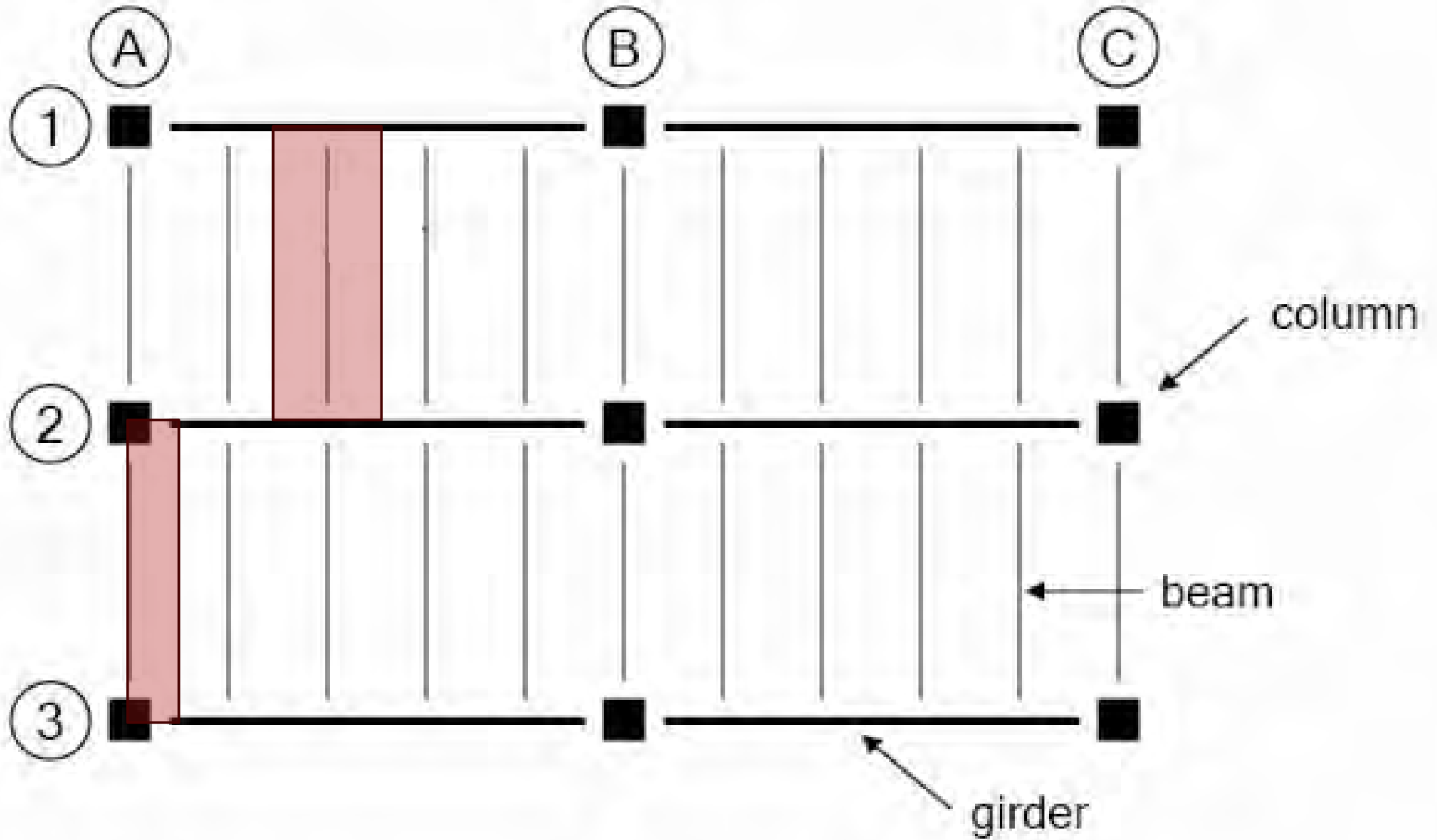
(c) cantiliver beam .

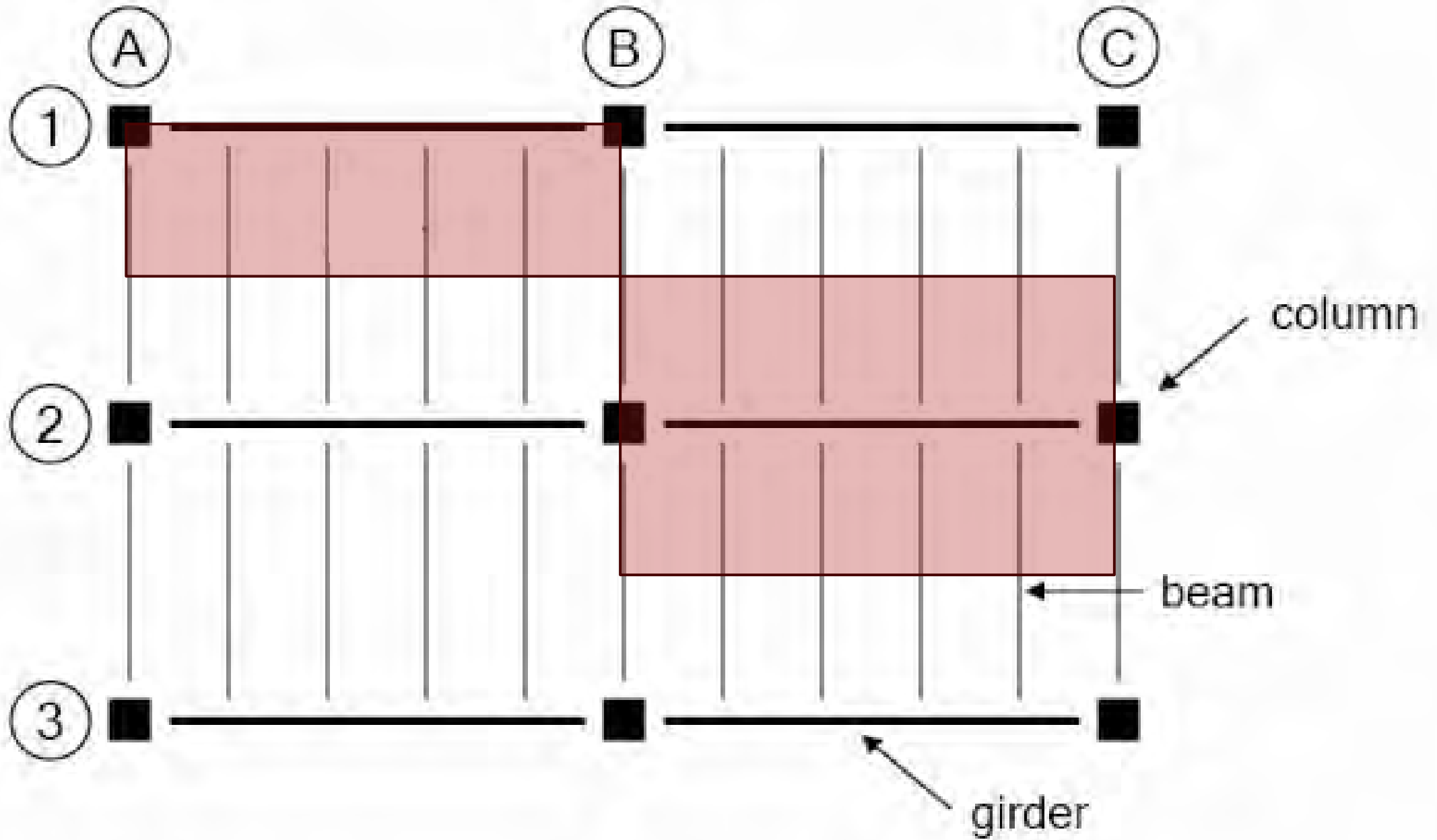
Tributary area/width

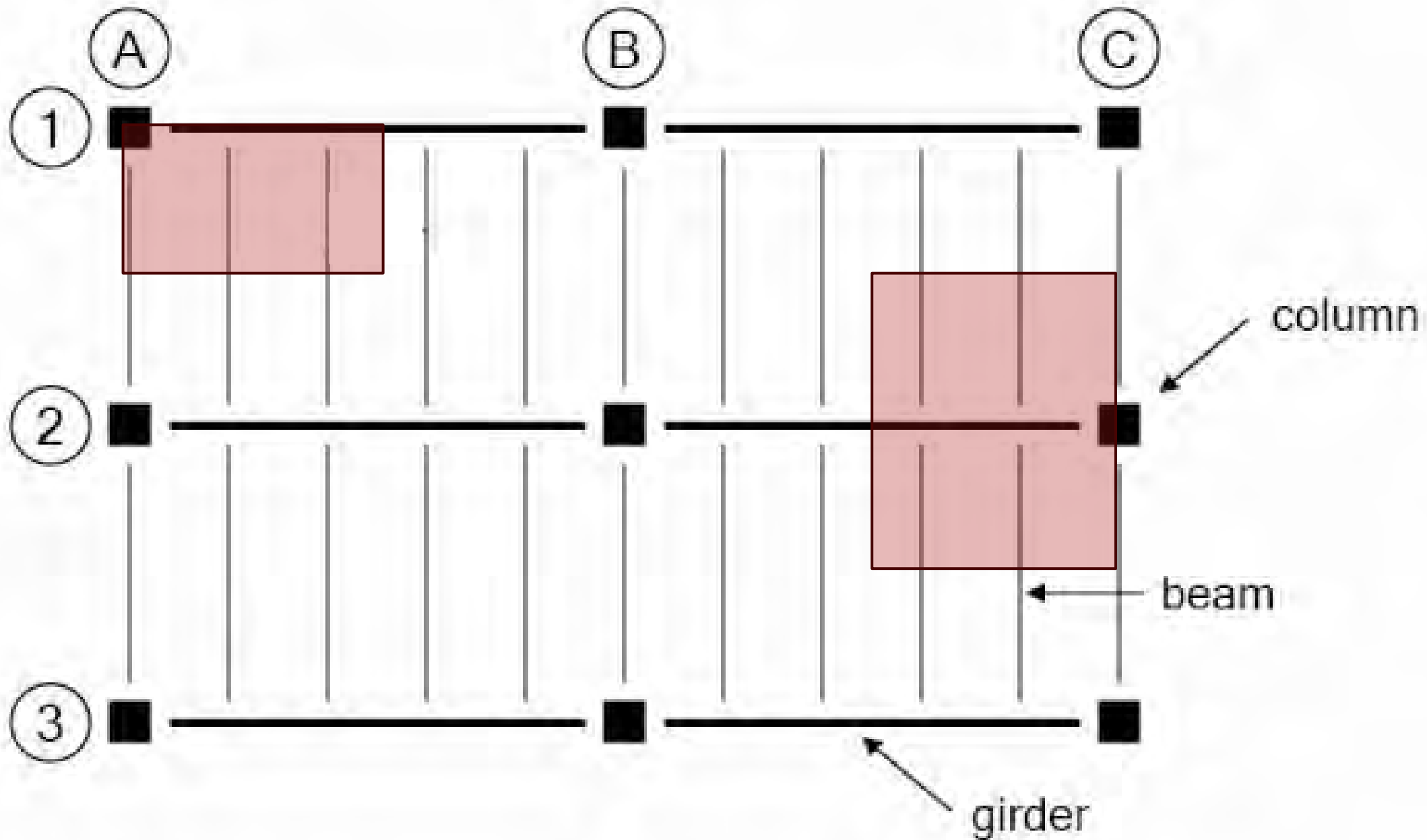
The area of a plane (floor, roof, or wall) that causes loading on a particular structural element.

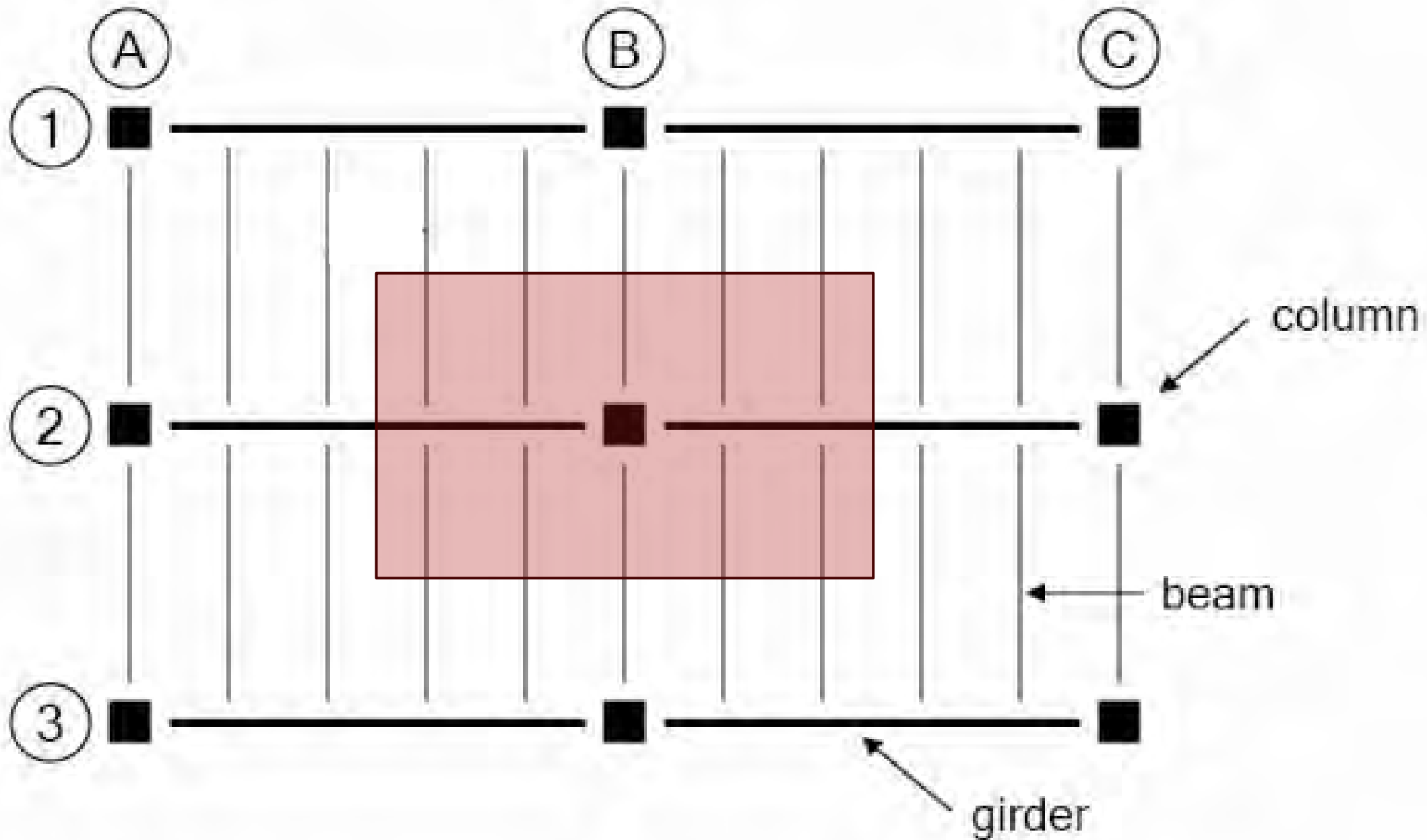










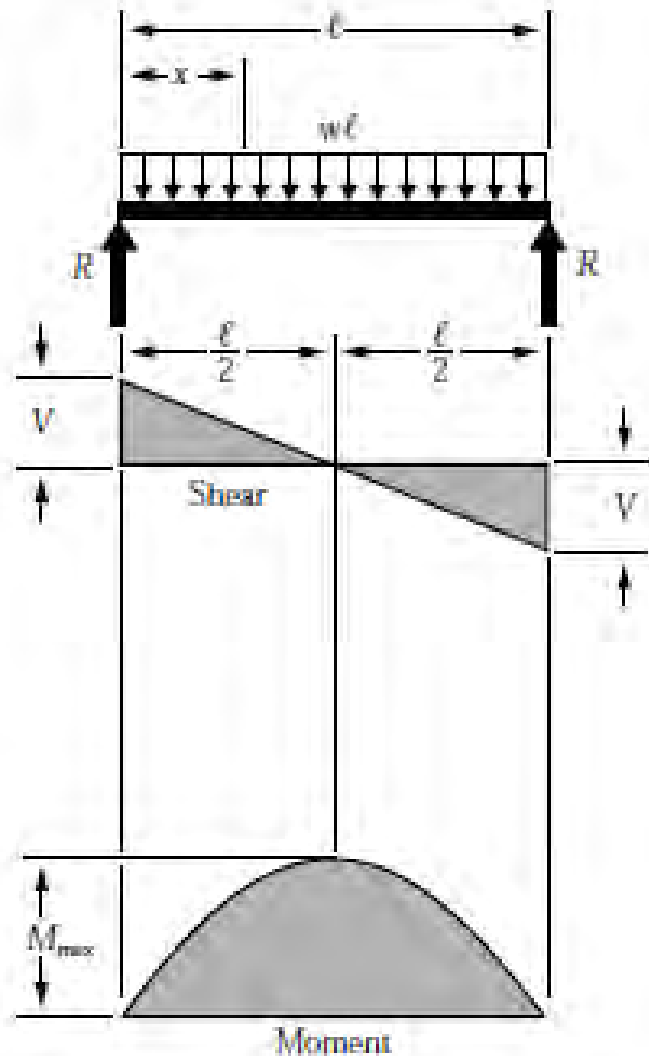


Minimum performance standards

- ▶ Stability
- ▶ Strength
- ▶ Service (Deflection)



Figure 1 Simple Beam – Uniformly Distributed Load



$$R = V \dots\dots\dots = \frac{wl}{2}$$

$$V_x \dots\dots\dots = w\left(\frac{l}{2} - x\right)$$

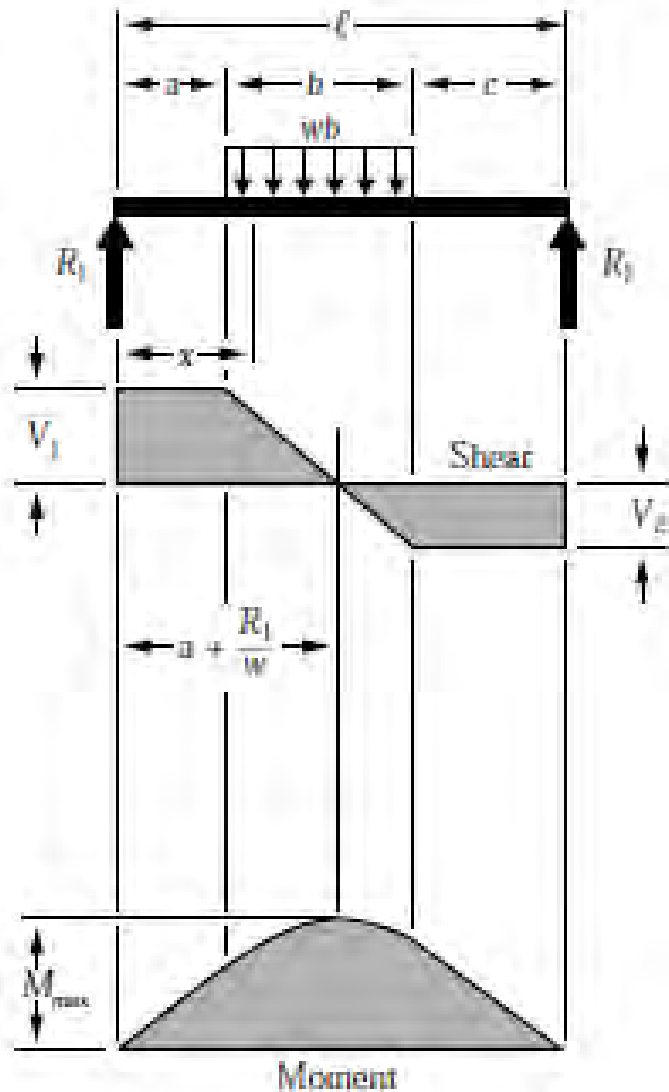
$$M_{max} \text{ (at center)} \dots\dots\dots = \frac{wl^2}{8}$$

$$M_x \dots\dots\dots = \frac{wx}{2}(l - x)$$

$$\Delta_{max} \text{ (at center)} \dots\dots\dots = \frac{5wl^4}{384 EI}$$

$$\Delta_x \dots\dots\dots = \frac{wx}{24 EI}(l^3 - 2lx^2 + x^3)$$

Figure 2 Simple Beam – Uniform Load Partially Distributed



$$R_1 = V_1 \text{ (max when } a < c) \dots \dots = \frac{wb}{2\ell}(2c + b)$$

$$R_2 = V_2 \text{ (max when } a > c) \dots \dots = \frac{wb}{2\ell}(2a + b)$$

$$V_x \text{ (when } x > a \text{ and } < (a + b)) \dots \dots = R_1 - w(x - a)$$

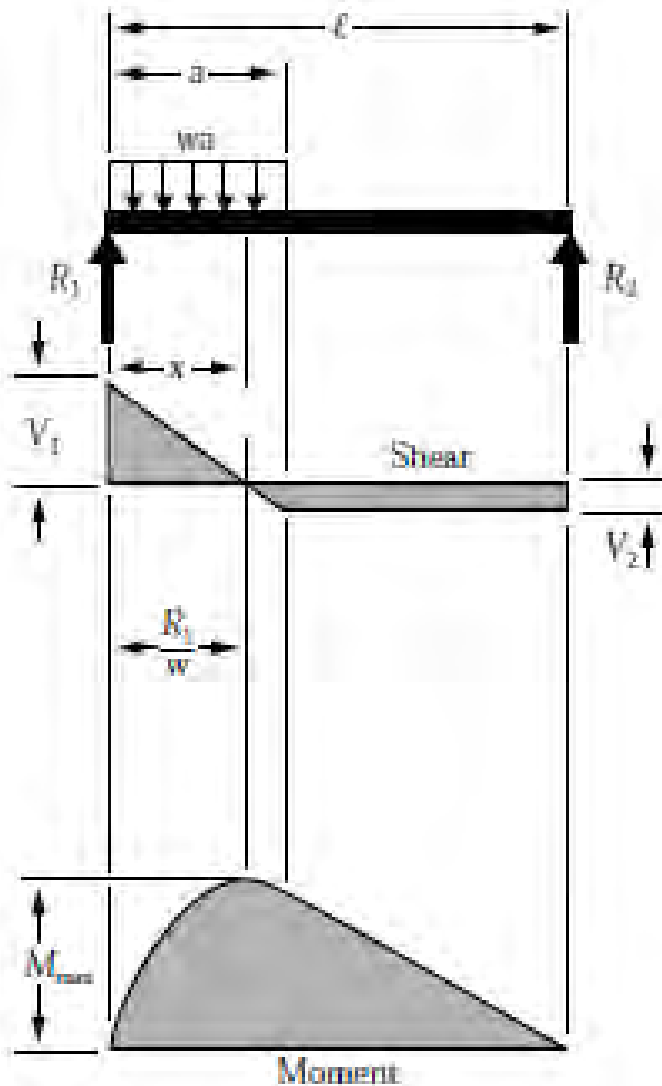
$$M_{max} \left(\text{at } x = a + \frac{R_1}{w} \right) \dots \dots = R_1 \left(a + \frac{R_1}{2w} \right)$$

$$M_x \text{ (when } x < a) \dots \dots \dots = R_1 x$$

$$M_x \text{ (when } x > a \text{ and } < (a + b)) \dots \dots = R_1 x - \frac{w}{2}(x - a)^2$$

$$M_x \text{ (when } x > (a + b)) \dots \dots \dots = R_2(\ell - x)$$

Figure 3 Simple Beam – Uniform Load Partially Distributed at One End



$$R_1 = V_1 \dots \dots \dots = \frac{wa}{2\ell} (2\ell - a)$$

$$R_2 = V_2 \dots \dots \dots = \frac{wa^2}{2\ell}$$

$$V_x \text{ (when } x < a) \dots \dots \dots = R_1 - wx$$

$$M_{\max} \left(\text{at } x = \frac{R_1}{w} \right) \dots \dots \dots = \frac{R_1^2}{2w}$$

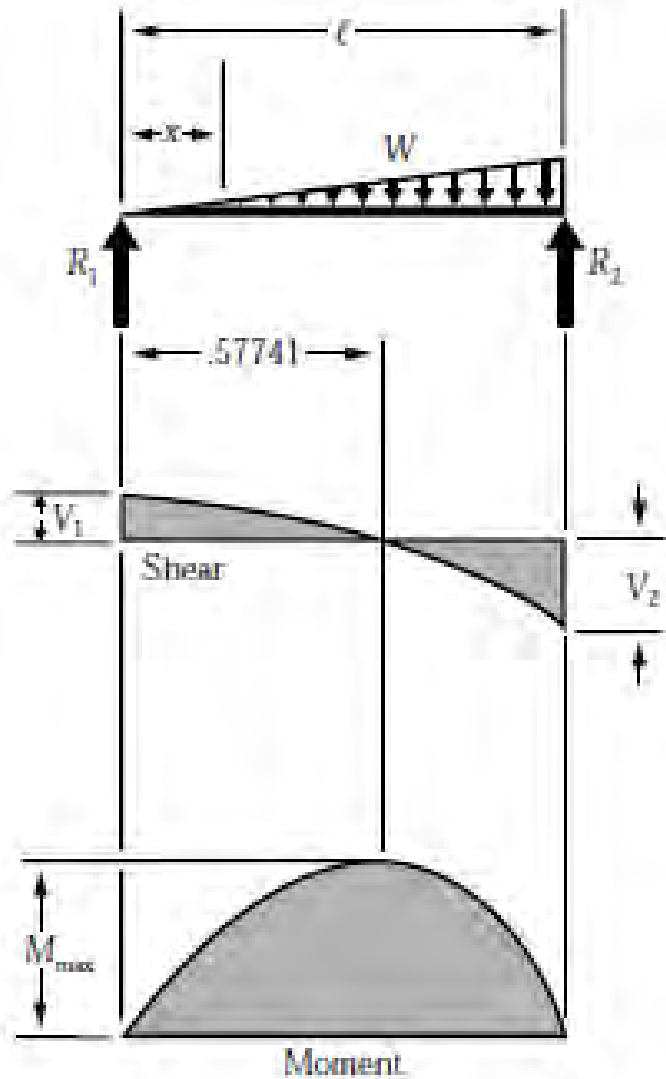
$$M_x \text{ (when } x < a) \dots \dots \dots = R_1 x - \frac{wx^2}{2}$$

$$M_x \text{ (when } x > a) \dots \dots \dots = R_2 (\ell - x)$$

$$\Delta_x \text{ (when } x < a) \dots \dots \dots = \frac{wx}{24 E I \ell} \left(a^2 (2\ell - a)^2 - 2ax^2 (2\ell - a) + \ell x^3 \right)$$

$$\Delta_x \text{ (when } x > a) \dots \dots \dots = \frac{wa^2 (\ell - x)}{24 E I \ell} (4x\ell - 2x^2 - a^2)$$

Figure 5 Simple Beam – Load Increasing Uniformly to One End



$$R_1 = V_1 \dots \dots \dots = \frac{W}{3}$$

$$R_2 = V_2 \dots \dots \dots = \frac{2W}{3}$$

$$V_x \dots \dots \dots = \frac{W}{3} - \frac{Wx^2}{l^2}$$

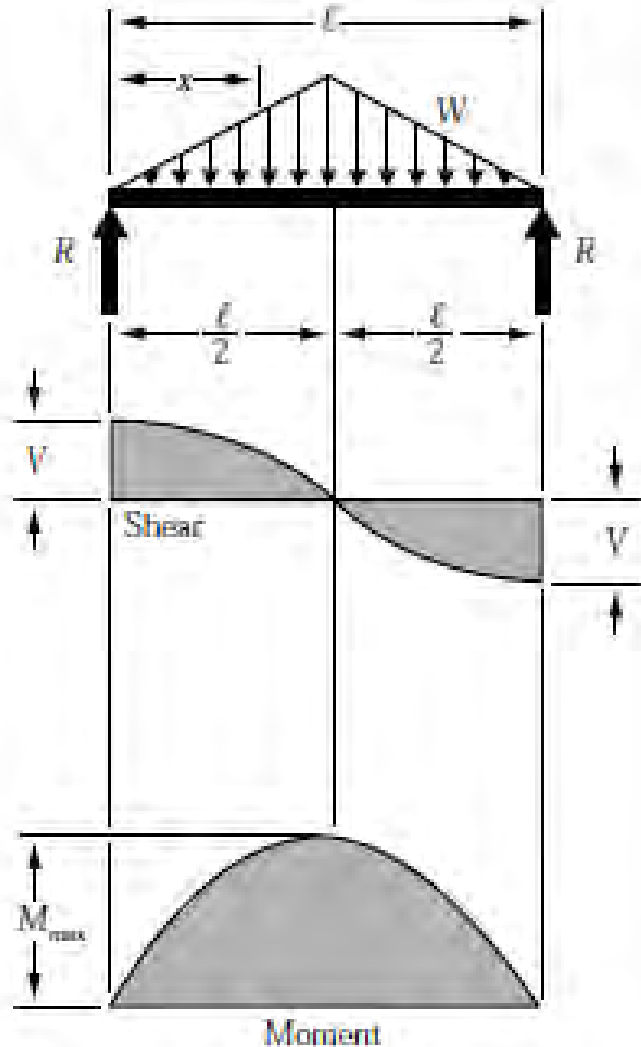
$$M_{max} \left(\text{at } x = \frac{l}{\sqrt{3}} = .5774l \right) \dots \dots = \frac{2Wl}{9\sqrt{3}} = .1283Wl$$

$$M_x \dots \dots \dots = \frac{Wx}{3l^2} (l^2 - x^2)$$

$$\Delta_{max} \left(\text{at } x = l \sqrt{1 - \sqrt{\frac{8}{15}}} = .5193l \right) \dots = .01304 \frac{Wl^3}{EI}$$

$$\Delta_x \dots \dots \dots = \frac{Wx}{180EI l^2} (3x^4 - 10l^2x^2 + 7l^4)$$

Figure 6 Simple Beam – Load Increasing Uniformly to Center



$$R = V \dots \dots \dots = \frac{W}{2}$$

$$V_x \left(\text{when } x < \frac{l}{2} \right) \dots \dots \dots = \frac{W}{2l^2} (l^2 - 4x^2)$$

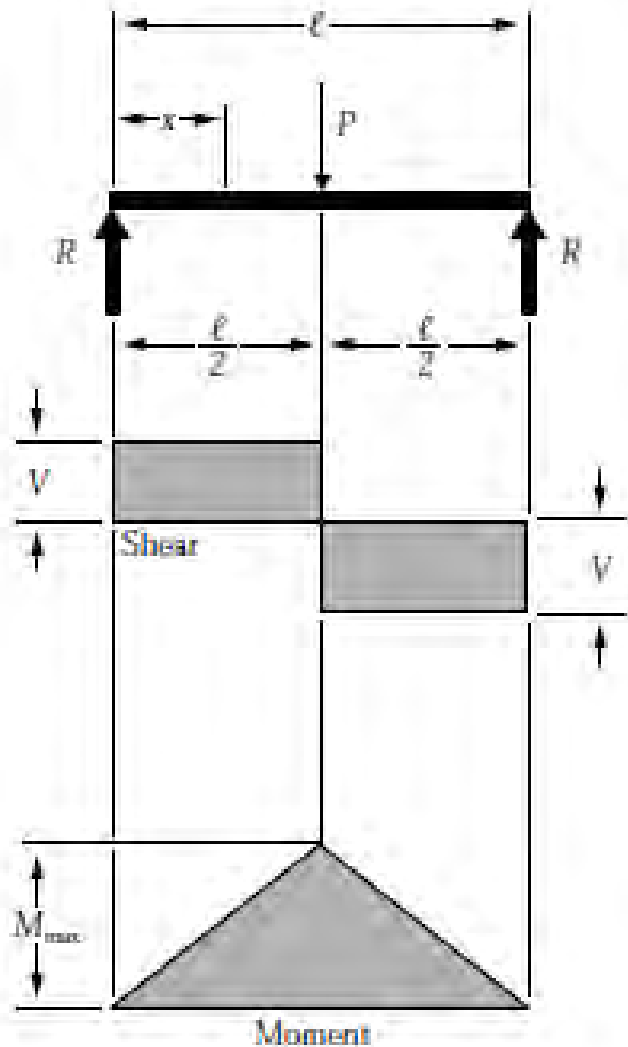
$$M_{\max} \text{ (at center)} \dots \dots \dots = \frac{Wl^3}{6}$$

$$M_x \left(\text{when } x < \frac{l}{2} \right) \dots \dots \dots = Wx \left(\frac{l}{2} - \frac{2x^2}{3l} \right)$$

$$\Delta_{\max} \text{ (at center)} \dots \dots \dots = \frac{Wl^4}{60EI}$$

$$\Delta_x \dots \dots \dots = \frac{Wx}{480EI l^2} (5l^2 - 4x^2)^2$$

Figure 7 Simple Beam – Concentrated Load at Center



$$R = V \dots \dots \dots = \frac{P}{2}$$

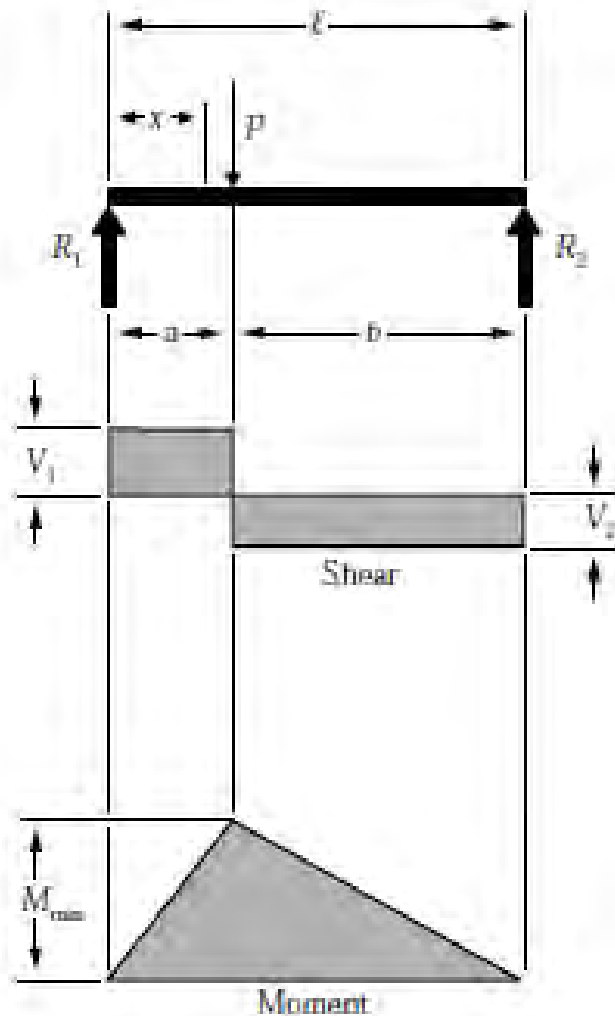
$$M_{max} \text{ (at point of load)} \dots \dots \dots = \frac{P\ell}{4}$$

$$M_x \left(\text{when } x < \frac{\ell}{2} \right) \dots \dots \dots = \frac{Px}{2}$$

$$\Delta_{max} \text{ (at point of load)} \dots \dots \dots = \frac{P\ell^3}{48EI}$$

$$\Delta_x \left(\text{when } x < \frac{\ell}{2} \right) \dots \dots \dots = \frac{Px}{48EI} (3\ell^2 - 4x^2)$$

Figure 8 Simple Beam – Concentrated Load at Any Point



$$R_1 = V_1 \text{ (max when } a < b) \dots\dots\dots = \frac{Pb}{l}$$

$$R_2 = V_2 \text{ (max when } a > b) \dots\dots\dots = \frac{Pa}{l}$$

$$M_{max} \text{ (at point of load) } \dots\dots\dots = \frac{Pab}{l}$$

$$M_x \text{ (when } x < a) \dots\dots\dots = \frac{Pbx}{l}$$

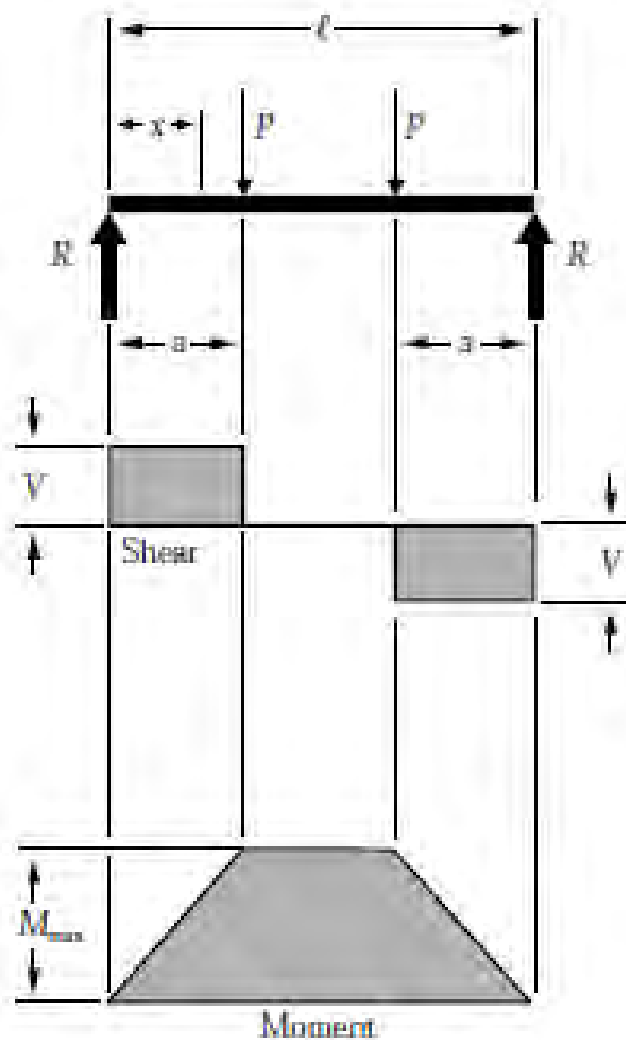
$$\Delta_{max} \left(\text{at } x = \sqrt{\frac{a(a+2b)}{3}} \text{ when } a > b \right) \dots\dots\dots = \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EI l}$$

$$\Delta_a \text{ (at point of load) } \dots\dots\dots = \frac{Pa^2b^2}{3EI l}$$

$$\Delta_x \text{ (when } x < a) \dots\dots\dots = \frac{Pbx}{6EI l} (l^2 - b^2 - x^2)$$

$$\Delta_x \text{ (when } x > a) \dots\dots\dots = \frac{Pa(l-x)}{6EI l} (2lx - x^2 - a^2)$$

Figure 9 Simple Beam – Two Equal Concentrated Loads Symmetrically Placed



$$R = V \text{ (at ends)} = P$$

$$M_{max} \text{ (between loads)} = Pa$$

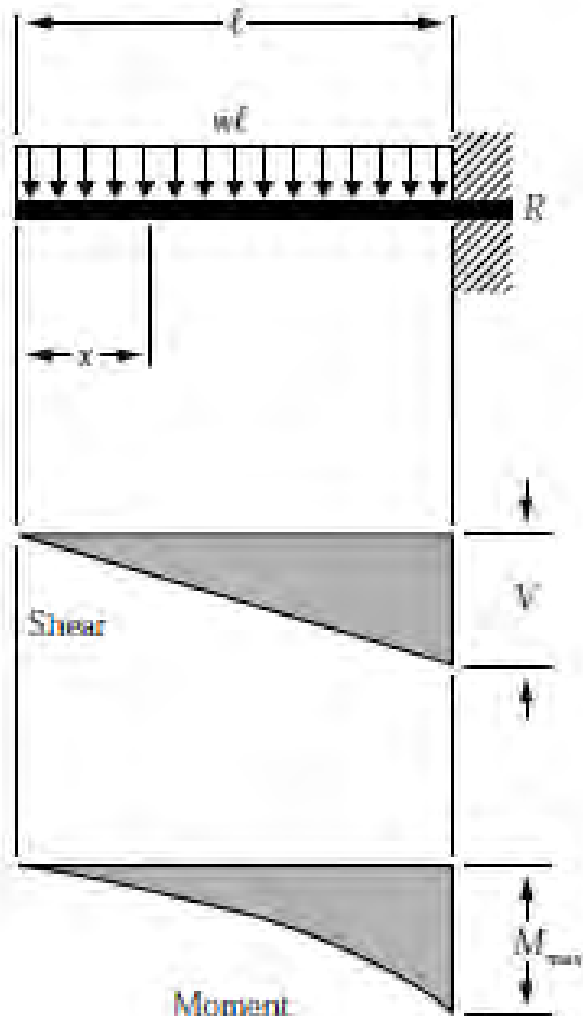
$$M_x \text{ (when } x < a) = Px$$

$$\Delta_{max} \text{ (at center)} = \frac{Pa}{24EI} (3l^2 - 4a^2)$$

$$\Delta_x \text{ (when } x < a) = \frac{Px}{6EI} (3la - 3a^2 - x^2)$$

$$\Delta_x \text{ (when } x > a \text{ and } < (l - a)) = \frac{Pa}{6EI} (3lx - 3x^2 - a^2)$$

Figure 12 Cantilever Beam – Uniformly Distributed Load



$$R = V \dots \dots \dots = wl$$

$$V_x \dots \dots \dots = wx$$

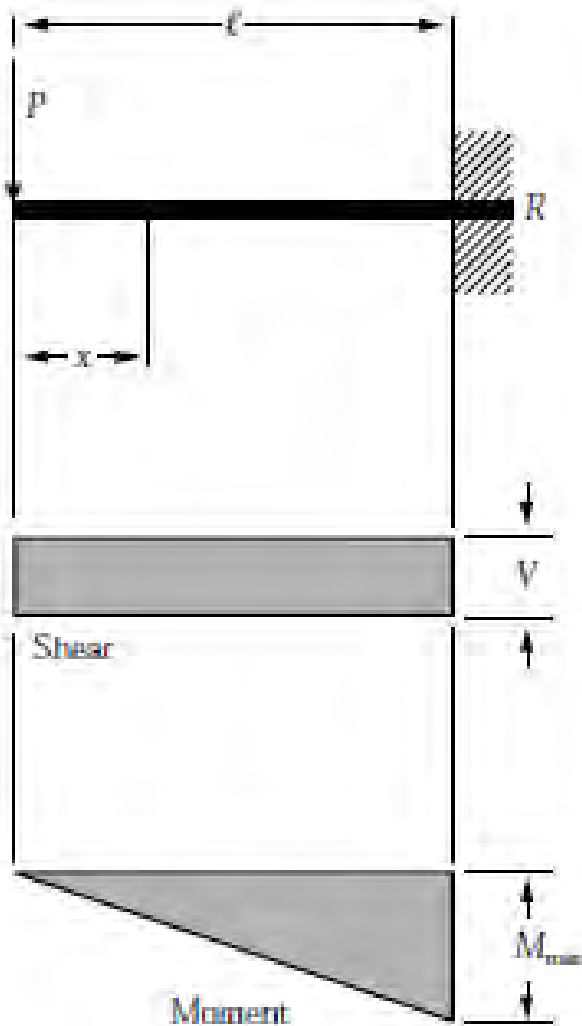
$$M_{max} \text{ (at fixed end)} \dots \dots \dots = \frac{wl^2}{2}$$

$$M_x \dots \dots \dots = \frac{wx^2}{2}$$

$$\Delta_{max} \text{ (at free end)} \dots \dots \dots = \frac{wl^4}{8EI}$$

$$\Delta_x \dots \dots \dots = \frac{w}{24EI} (x^4 - 4l^3x + 3l^4)$$

Figure 13 Cantilever Beam – Concentrated Load at Free End



$$R = V \dots \dots \dots = P$$

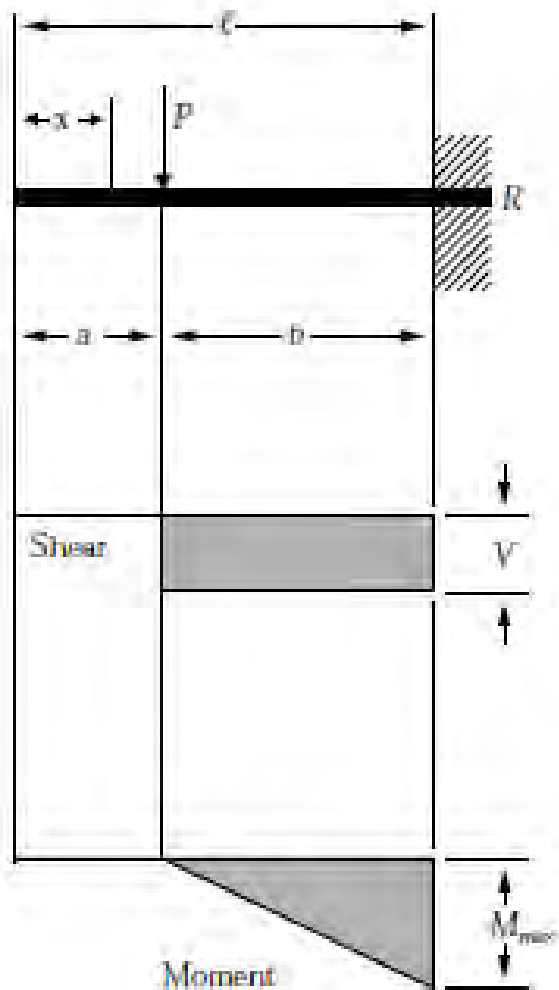
$$M_{max} \text{ (at fixed end)} \dots \dots \dots = Pl$$

$$M_x \dots \dots \dots = Px$$

$$\Delta_{max} \text{ (at free end)} \dots \dots \dots = \frac{Pl^3}{3EI}$$

$$\Delta_x \dots \dots \dots = \frac{P}{6EI} (2l^3 - 3l^2x + x^3)$$

Figure 14 Cantilever Beam – Concentrated Load at Any Point



$$R = V \dots \dots \dots = P$$

$$M_{max} \text{ (at fixed end)} \dots \dots \dots = Pb$$

$$M_x \text{ (when } x > a) \dots \dots \dots = P(x - a)$$

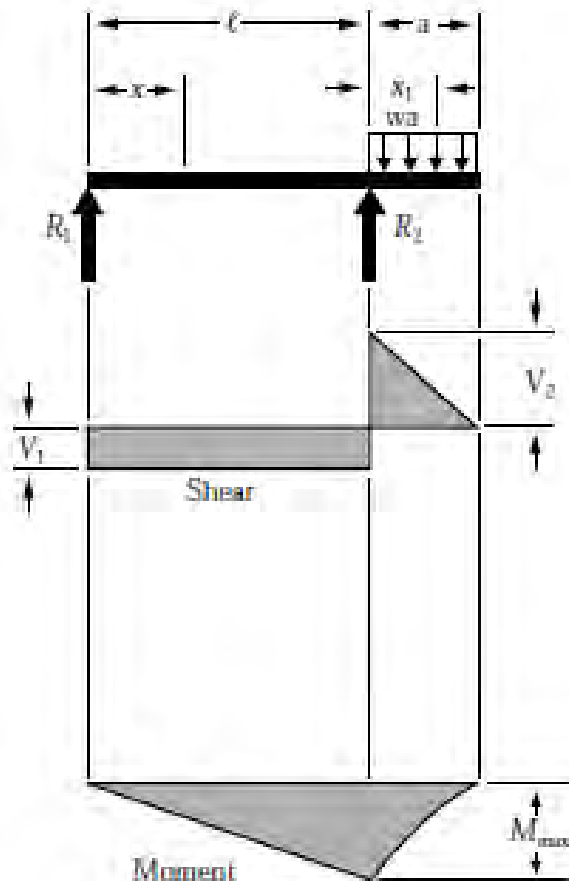
$$\Delta_{max} \text{ (at free end)} \dots \dots \dots = \frac{Pb^2}{6EI} (3l - b)$$

$$\Delta_x \text{ (at point of load)} \dots \dots \dots = \frac{Pb^3}{3EI}$$

$$\Delta_x \text{ (when } x < a) \dots \dots \dots = \frac{Pb^2}{6EI} (3l - 3x - b)$$

$$\Delta_x \text{ (when } x > a) \dots \dots \dots = \frac{P(l - x)^2}{6EI} (3b - l + x)$$

Figure 19 Beam Overhanging One Support – Uniformly Distributed Load on Overhang



$$R_1 = V_1 = \dots = \frac{wa^2}{2l}$$

$$R_2 = V_1 + V_2 = \dots = \frac{wa}{2l}(2l + a)$$

$$V_2 = \dots = wa$$

$$V_{x_1} \text{ (for overhang)} = \dots = w(a - x_1)$$

$$M_{\max} \text{ (at } R_2) = \dots = \frac{wa^2}{2}$$

$$M_x \text{ (between supports)} = \dots = \frac{wa^2x}{2l}$$

$$M_{x_1} \text{ (for overhang)} = \dots = \frac{w}{2}(a - x_1)^2$$

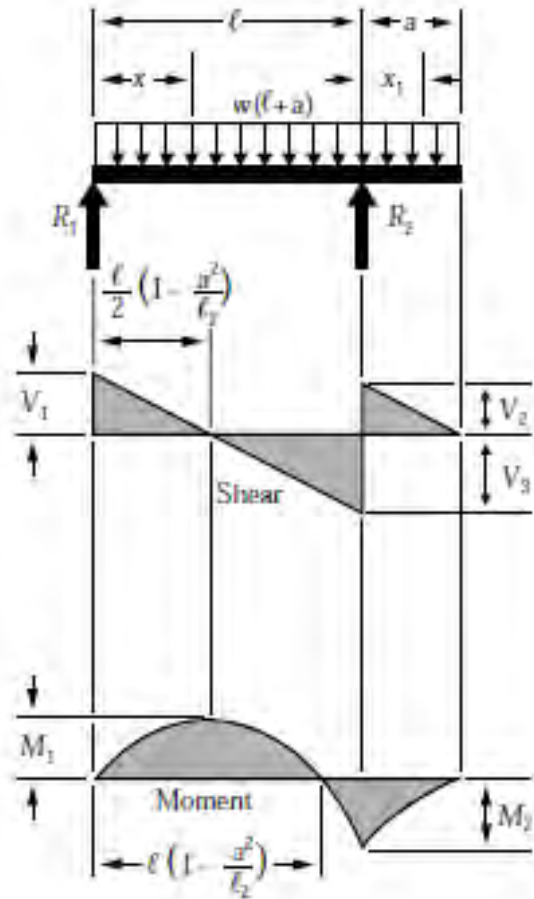
$$\Delta_{\max} \left(\text{between supports at } x = \frac{l}{\sqrt{3}} \right) = \frac{wa^2l^2}{18\sqrt{3}EI} = .03208 \frac{wa^2l^2}{EI}$$

$$\Delta_{\max} \text{ (for overhang at } x_1 = a) = \dots = \frac{wa^3}{24EI}(4l + 3a)$$

$$\Delta_x \text{ (between supports)} = \dots = \frac{wa^2x}{12EIl}(l^2 - x^2)$$

$$\Delta_{x_1} \text{ (for overhang)} = \dots = \frac{wx_1}{24EI}(4a^2l + 6a^2x_1 - 4ax_1^2 + x_1^3)$$

Figure 18 Beam Overhanging One Support – Uniformly Distributed Load



$$R_1 = V_1 \dots\dots\dots = \frac{w}{2\ell}(\ell^2 - a^2)$$

$$R_2 = V_2 + V_3 \dots\dots\dots = \frac{w}{2\ell}(\ell + a)^2$$

$$V_2 \dots\dots\dots = wa$$

$$V_3 \dots\dots\dots = \frac{w}{2\ell}(\ell^2 + a^2)$$

$$V_x \text{ (between supports)} \dots\dots\dots = R_1 - wx$$

$$V_{x_1} \text{ (for overhang)} \dots\dots\dots = w(a - x_1)$$

$$M_1 \left(\text{at } x = \frac{\ell}{2} \left[1 - \frac{a^2}{\ell^2} \right] \right) \dots\dots\dots = \frac{w}{8\ell^2}(\ell + a)^2(\ell - a)^2$$

$$M_2 \text{ (at } R_2) \dots\dots\dots = \frac{wa^2}{2}$$

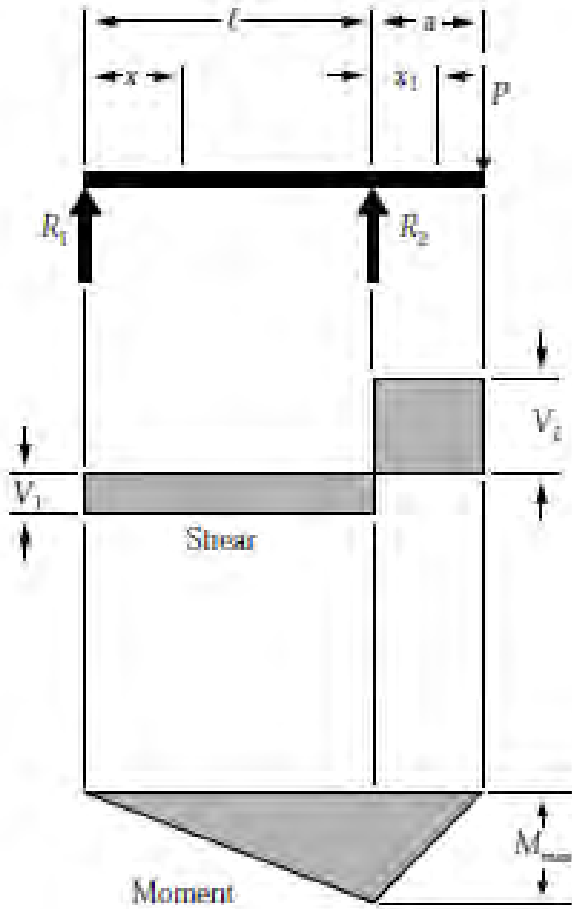
$$M_x \text{ (between supports)} \dots\dots\dots = \frac{wx}{2\ell}(\ell^2 - a^2 - x\ell)$$

$$M_{x_1} \text{ (for overhang)} \dots\dots\dots = \frac{w}{2}(a - x_1)^2$$

$$\Delta_x \text{ (between supports)} \dots\dots\dots = \frac{wx}{24EI\ell}(\ell^4 - 2\ell^2x^2 + \ell x^3 - 2a^2\ell^2 + 2a^2x^2)$$

$$\Delta_{x_1} \text{ (for overhang)} \dots\dots\dots = \frac{wx_1}{24EI}(4a^2\ell - \ell^3 + 6a^2x_1 - 4ax_1^2 + x_1^3)$$

Figure 20 Beam Overhanging One Support – Concentrated Load at End of Overhang



$$R_1 = V_1 \dots \dots \dots = \frac{Pa}{l}$$

$$R_2 = V_1 + V_2 \dots \dots \dots = \frac{P}{l}(l + a)$$

$$V_2 \dots \dots \dots = P$$

$$M_{\max} \text{ (at } R_2) \dots \dots \dots = Pa$$

$$M_x \text{ (between supports)} \dots \dots \dots = \frac{Pax}{l}$$

$$M_{x_1} \text{ (for overhang)} \dots \dots \dots = P(a - x_1)$$

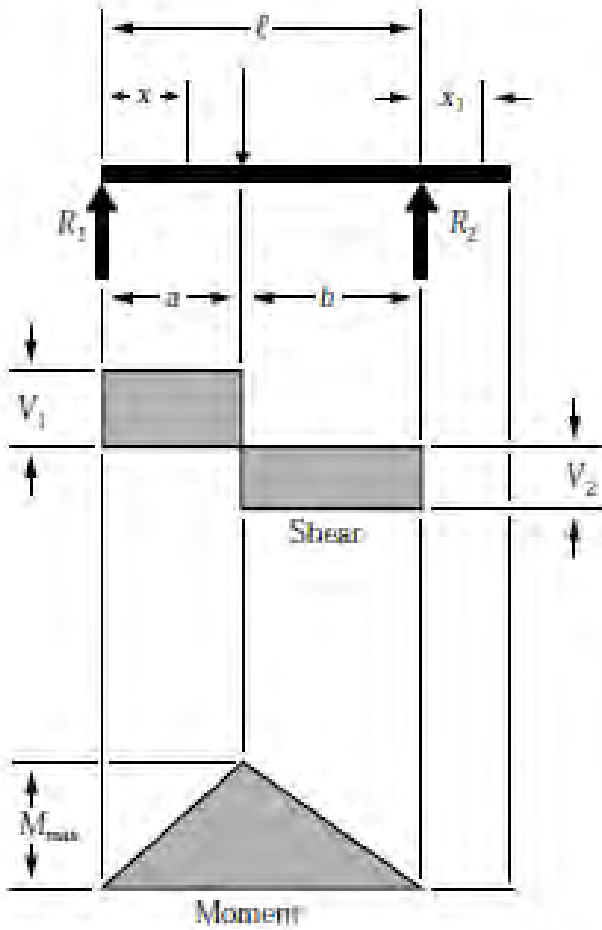
$$\Delta_{\max} \left(\text{between supports at } x = \frac{l}{\sqrt{3}} \right) = \frac{Pal^2}{9\sqrt{3}EI} = .06415 \frac{Pal^2}{EI}$$

$$\Delta_{\max} \text{ (for overhang at } x_1 = a) \dots \dots \dots = \frac{Pa^2}{3EI}(l + a)$$

$$\Delta_x \text{ (between supports)} \dots \dots \dots = \frac{Pax}{6EI}(l^2 - x^2)$$

$$\Delta_{x_1} \text{ (for overhang)} \dots \dots \dots = \frac{Px_1}{6EI}(2al + 3ax_1 - x_1^2)$$

Figure 21 Beam Overhanging One Support – Concentrated Load at Any Point Between Supports



$$R_1 = V_1 \text{ (max when } a < b) \dots \dots \dots = \frac{Pb}{l}$$

$$R_2 = V_2 \text{ (max when } a > b) \dots \dots \dots = \frac{Pa}{l}$$

$$M_{\max} \text{ (at point of load) } \dots \dots \dots = \frac{Pab}{l}$$

$$M_x \text{ (when } x < a) \dots \dots \dots = \frac{Pbx}{l}$$

$$\Delta_{\max} \left(\text{at } x = \sqrt{\frac{a(a+2b)}{3}} \text{ when } a > b \right) \dots \dots \dots = \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EI\ell}$$

$$\Delta_a \text{ (at point of load) } \dots \dots \dots = \frac{Pa^2b^2}{3EI\ell}$$

$$\Delta_x \text{ (when } x < a) \dots \dots \dots = \frac{Pbx}{6EI\ell} (\ell^2 - b^2 - x^2)$$

$$\Delta_x \text{ (when } x > a) \dots \dots \dots = \frac{Pa(\ell - x)}{6EI\ell} (2\ell x - x^2 - a^2)$$

$$\Delta_{x_1} \dots \dots \dots = \frac{Pabx_1}{6EI\ell} (\ell + a)$$



Deflection

TABLE 1604.3
DEFLECTION LIMITS^{a, b, c, h, i}

| CONSTRUCTION | L or L_r | S or W^f | $D + L^{d, g}$ |
|--------------------------------------|--------------|--------------|----------------|
| Roof members: ^e | | | |
| Supporting plaster or stucco ceiling | $l/360$ | $l/360$ | $l/240$ |
| Supporting nonplaster ceiling | $l/240$ | $l/240$ | $l/180$ |
| Not supporting ceiling | $l/180$ | $l/180$ | $l/120$ |
| Floor members | $l/360$ | — | $l/240$ |
| Exterior walls: | | | |
| With plaster or stucco finishes | — | $l/360$ | — |
| With other brittle finishes | — | $l/240$ | — |
| With flexible finishes | — | $l/120$ | — |
| Interior partitions: ^b | | | |
| With plaster or stucco finishes | $l/360$ | — | — |
| With other brittle finishes | $l/240$ | — | — |
| With flexible finishes | $l/120$ | — | — |
| Farm buildings | — | — | $l/180$ |
| Greenhouses | — | — | $l/120$ |



Superposition

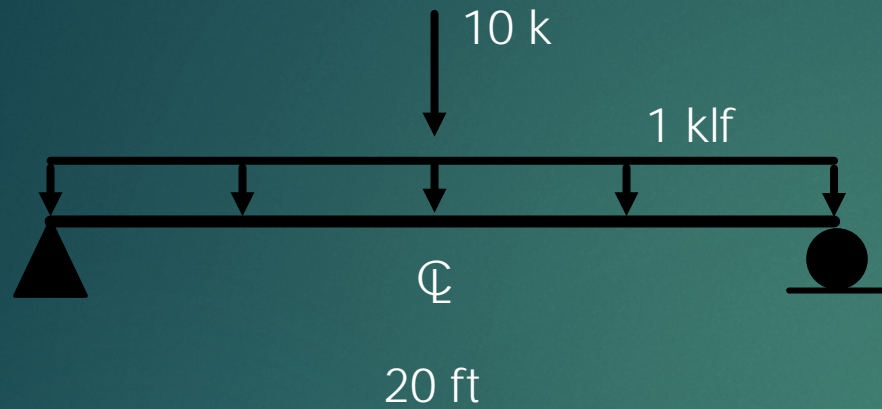
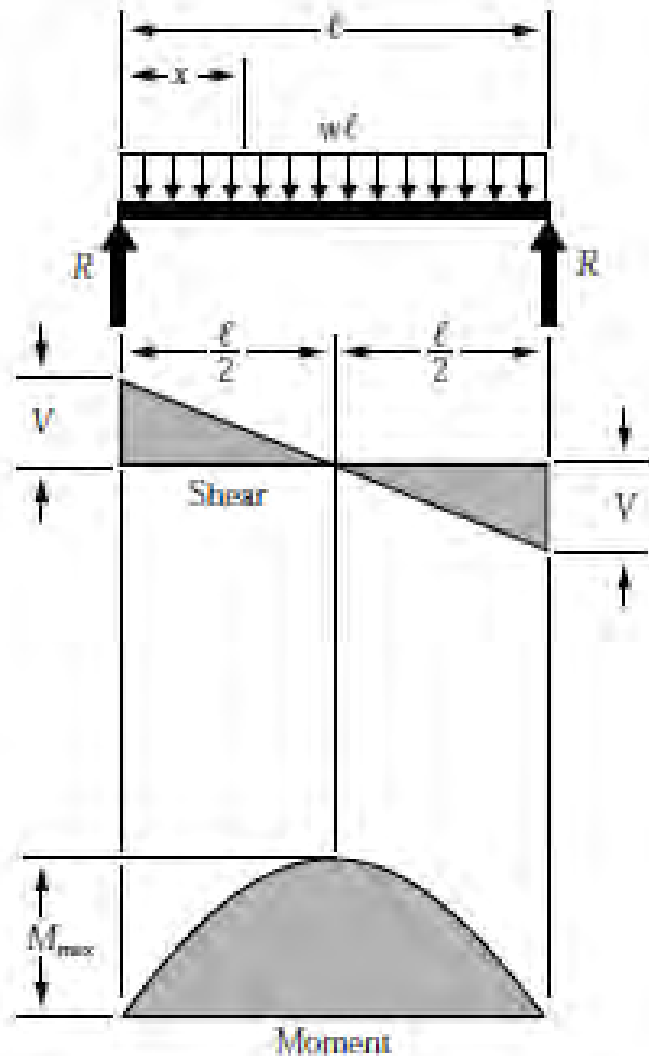


Figure 1 Simple Beam – Uniformly Distributed Load



$$R = V \dots \dots \dots = \frac{wl}{2}$$

$$V_x \dots \dots \dots = w\left(\frac{l}{2} - x\right)$$

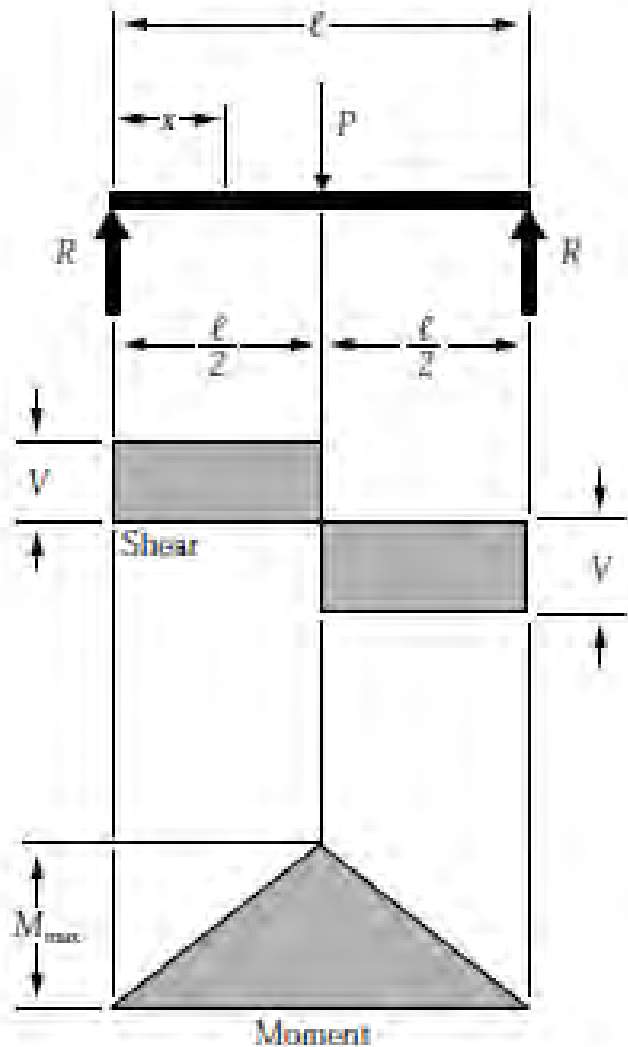
$$M_{max} \text{ (at center)} \dots \dots \dots = \frac{wl^2}{8}$$

$$M_x \dots \dots \dots = \frac{wx}{2}(l - x)$$

$$\Delta_{max} \text{ (at center)} \dots \dots \dots = \frac{5wl^4}{384 EI}$$

$$\Delta_x \dots \dots \dots = \frac{wx}{24 EI}(l^3 - 2lx^2 + x^3)$$

Figure 7 Simple Beam – Concentrated Load at Center



$$R = V \dots \dots \dots = \frac{P}{2}$$

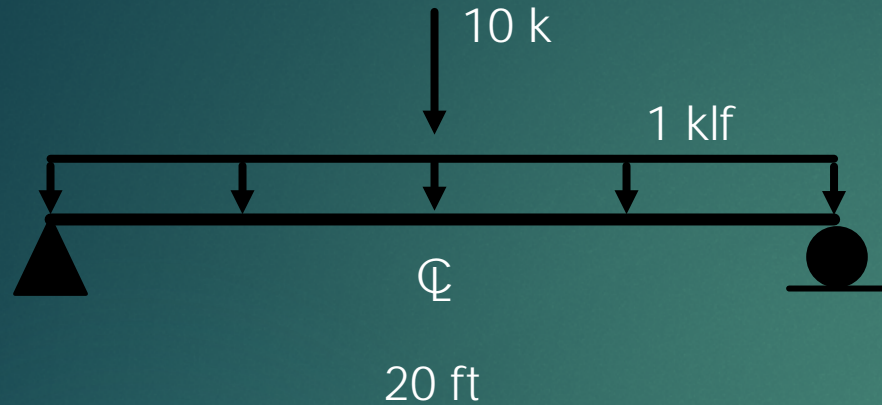
$$M_{max} \text{ (at point of load)} \dots \dots \dots = \frac{P\ell}{4}$$

$$M_x \left(\text{when } x < \frac{\ell}{2} \right) \dots \dots \dots = \frac{Px}{2}$$

$$\Delta_{max} \text{ (at point of load)} \dots \dots \dots = \frac{P\ell^3}{48EI}$$

$$\Delta_x \left(\text{when } x < \frac{\ell}{2} \right) \dots \dots \dots = \frac{Px}{48EI} (3\ell^2 - 4x^2)$$

Superposition



$$V_{max} = \frac{Wl}{2} = \frac{(1 \text{ klf})(20 \text{ ft})}{2} = 10\text{k}$$

$$V_{max} = \frac{P}{2} = \frac{(10 \text{ k})}{2} = 5\text{k}$$

$$V_{max} = 10\text{k} + 5\text{k} = 15\text{k}$$

$$M_{max} = \frac{Wl^2}{8} = \frac{(1 \text{ klf})(20\text{ft})^2}{8} = 50\text{ft}\cdot\text{k}$$

$$M_{max} = \frac{Pl}{4} = \frac{(10 \text{ k})(20\text{ft})}{4} = 50\text{ft}\cdot\text{k}$$

$$M_{max} = 50\text{ft} - k + 50\text{ft} - k = 100\text{ft} - k$$

Loads on Buildings

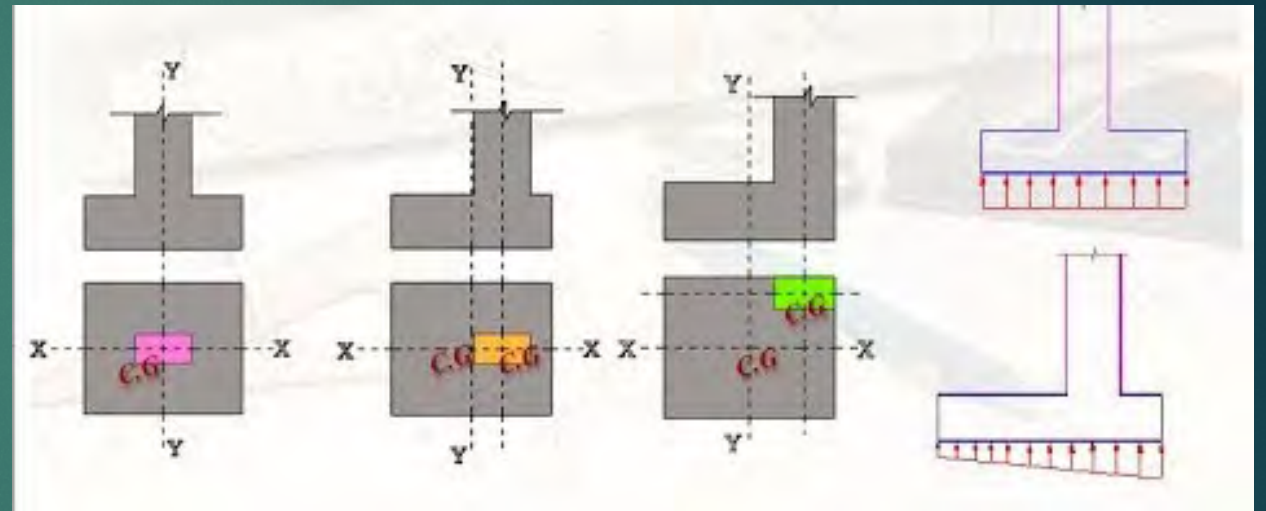


Concentrated Load v. distributed loads



Concentric v. eccentric Loading

- ▶ Concentric loading – the load passes through the center of gravity of the member
- ▶ Eccentric Loading – the load passes at some distance (eccentricity) from the center of gravity of the member



Gravity Loads

- ▶ Force due the gravitational force (acceleration due to gravity) of the earth, typically acting toward the center of the earth.
- ▶ $F=MA$,
 - M is the mass of the object
 - A is the Acceleration due to gravity 32.2 f/s^2 or 1 g or 9.81 m/s^2



Gravity Loads

- ▶ Dead Load - buildings self-weight, permanently attached fixtures and coverings, mechanical equipment, very well known, little change over time
- ▶ Live Load - transient in nature, not as well known as dead load, changes over time, people and furniture
- ▶ Snow Load - load from frozen precipitation
- ▶ Rain Load – load from precipitation



Lateral

- ▶ Wind Load – pressure on the building from the movement of air, resulting from differences in atmospheric pressures
- ▶ Seismic/Earthquake Load – the response of the building to ground movement and momentum, effected by building mass, soil properties, structural configuration, structural material, acceleration and velocity of the earths movement
- ▶ Live load – movement of people, vehicles, etc.



Load Duration

- ▶ Static Load – applied slowly and typically present over a long period of time like file storage in an office building
- ▶ Dynamic or Impact Load – applied quickly, can have an increased load effect in excess of a factor of two (2) like the effect of a motor vehicle hitting a building



Loads

Forces or other actions that result from the weight of building materials, occupants and their possessions, environmental effects, differential movement and restrained dimensional changes. Permanent loads are those loads in which variations over time are rare or of small magnitude, such as dead loads. All other loads are variable loads (see “Nominal loads”).



Dead Load

The weight of materials of construction incorporated into the building, including but not limited to walls, floors, roofs, ceilings, stairways, built-in partitions, finishes, cladding and other similarly incorporated architectural and structural items, and the weight of fixed service equipment, such as cranes, plumbing stacks and risers, electrical feeders, heating, ventilating and air-conditioning systems and automatic sprinkler systems.



Table C3.1-1a Minimum Design Dead Loads (psf)^a

| Component | Load (psf) |
|---|------------|
| CEILINGS | |
| Acoustical fiberboard | 1 |
| Gypsum board (per 1/8-in. thickness) | 0.55 |
| Mechanical duct allowance | 4 |
| Plaster on tile or concrete | 5 |
| Plaster on wood lath | 8 |
| Suspended steel channel system | 2 |
| Suspended metal lath and cement plaster | 15 |
| Suspended metal lath and gypsum plaster | 10 |
| Wood furring suspension system | 2.5 |
| COVERINGS, ROOF, AND WALL | |
| Asbestos-cement shingles | 4 |
| Asphalt shingles | 2 |
| Cement tile | 16 |
| Clay tile (for mortar add 10 psf) | |
| Book tile, 2-in. | 12 |
| Book tile, 3-in. | 20 |
| Ludowici | 10 |
| Roman | 12 |
| Spanish | 19 |
| Composition: | |
| Three-ply ready roofing | 1 |
| Four-ply felt and gravel | 5.5 |
| Five-ply felt and gravel | 6 |

Table C3.1-1a (Continued)

| Component | Load (psf) | | |
|---|----------------------|----------------------|----------------------|
| Slate (per mm thickness) | 15 | | |
| Solid flat tile on 1-in. mortar base | 23 | | |
| Subflooring, 3/4-in. | 3 | | |
| Terrazzo (1-1/2-in.) directly on slab | 19 | | |
| Terrazzo (1-in.) on stone-concrete fill | 32 | | |
| Terrazzo (1-in.), 2-in. stone concrete | 32 | | |
| Wood block (3-in.) on mastic, no fill | 10 | | |
| Wood block (3-in.) on 1/2-in. mortar base | 16 | | |
| FLOORS, WOOD-JOIST (NO PLASTER) | | | |
| DOUBLE WOOD FLOOR | | | |
| Joint sizes (in.) | 12-in. spacing (psf) | 16-in. spacing (psf) | 24-in. spacing (psf) |
| 2 × 6 | 6 | 5 | 5 |
| 2 × 8 | 6 | 6 | 5 |
| 2 × 10 | 7 | 6 | 6 |
| 2 × 12 | 8 | 7 | 6 |
| FRAME PARTITIONS | | | |
| Movable steel partitions | | | 4 |
| Wood or steel studs, 1/2-in. gypsum board each side | | | 8 |
| Wood studs, 2 × 4, unplastered | | | 4 |
| Wood studs, 2 × 4, plastered one side | | | 12 |
| Wood studs, 2 × 4, plastered two sides | | | 20 |
| FRAME WALLS | | | |
| Exterior stud walls: | | | |
| 2 × 4 @ 16-in., 5/8-in. gypsum, insulated, 3/8-in. siding | | | 11 |
| 2 × 6 @ 16-in., 5/8-in. gypsum, insulated, 3/8-in. siding | | | 12 |
| Exterior stud walls with brick veneer | | | 48 |
| Windows, glass, frame, and sash | | | 8 |

Live Load

A load produced by the use and occupancy of the building or other structure that does not include construction or environmental loads such as wind load, snow load, rain load, earthquake load, flood load or dead load.



Live Load

- ▶ **1607.2 Loads not specified.** For occupancies or uses not designated in Table 1607.1, the live load shall be determined in accordance with a method approved by the building official.
- ▶ **1607.3 Uniform live loads.** The live loads used in the design of buildings and other structures shall be the maximum loads expected by the intended use or occupancy but shall not be less than the minimum uniformly distributed live loads given in Table 1607.1.



Live Load

1607.4 Concentrated live loads. Floors, roofs and other similar surfaces shall be designed to support the uniformly distributed live loads prescribed in Section 1607.3 or the concentrated live loads, given in Table 1607.1, whichever produces the greater load effects. Unless otherwise specified, the indicated concentration shall be assumed to be uniformly distributed over an area of 2-1/2 feet by 2-1/2 feet (762 mm by 762 mm) and shall be located so as to produce the maximum load effects in the structural members.



Live Load

1607.5 Partition loads. In office buildings and in other buildings where partition locations are subject to change, provisions for partition weight shall be made, whether or not partitions are shown on the construction documents, unless the specified live load is 80 psf (3.83 kN/m²) or greater. The partition load shall be not less than a uniformly distributed live load of 15 psf (0.72 kN/m²).



**TABLE 1607.1
MINIMUM UNIFORMLY DISTRIBUTED LIVE LOADS, L_u ,
AND MINIMUM CONCENTRATED LIVE LOADS³**

| OCCUPANCY OR USE | UNIFORM (psf) | CONCENTRATED (pounds) |
|---|---|--------------------------|
| 1. Apartments (see residential) | — | — |
| 2. Access floor systems | | |
| Office use | 50 | 2,000 |
| Computer use | 100 | 2,000 |
| 3. Armories and drill rooms | 150 ^a | — |
| 4. Assembly areas | | |
| Fixed seats (fastened to floor) | 60 ^a | |
| Follow spot, projections and control rooms | 50 | |
| Lobbies | 100 ^a | — |
| Movable seats | 100 ^a | |
| Stage floors | 150 ^a | |
| Platforms (assembly) | 100 ^a | |
| Other assembly areas | 100 ^a | |
| 5. Balconies and decks ^b | 1.5 times the live load for the area served, not required to exceed 100 | — |
| 6. Catwalks | 40 | 300 |
| 7. Cornices | 60 | — |



Live Load, Roof

A load on a roof produced:

1. During maintenance by workers, equipment and materials;
2. During the life of the structure by movable objects such as planters or other similar small decorative appurtenances that are not occupancy related; or
3. By the use and occupancy of the roof such as for roof gardens or assembly areas.

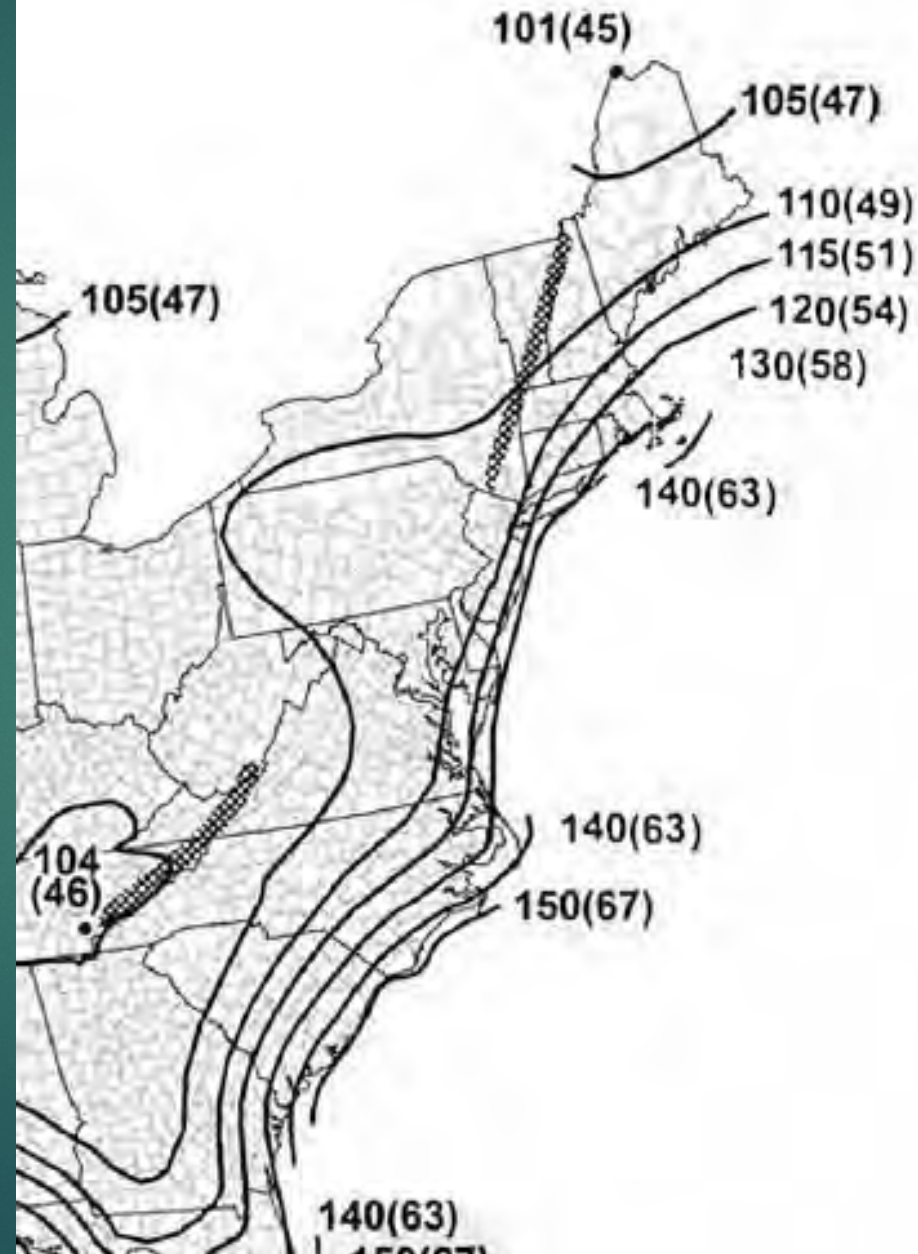


Snow Load

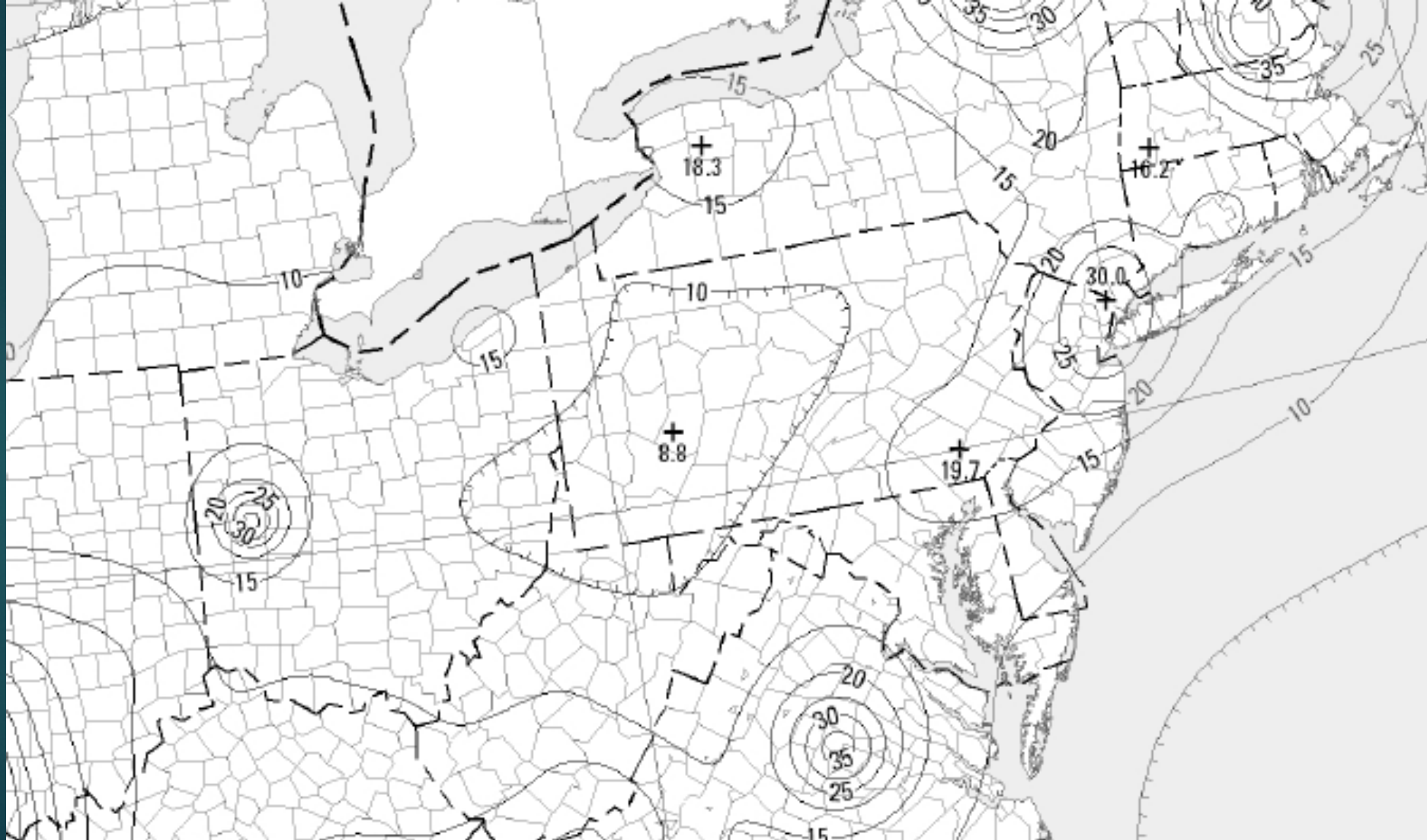
- ▶ **GROUND SNOW LOAD:** The site-specific weight of the accumulated snow at the ground level used to develop roof snow loads on the structure. It generally has a 50-year mean recurrence interval.
- ▶ **MINIMUM SNOW LOAD:** Snow load on low sloped roofs, including the roof snow load immediately after a single snow storm without wind.
- ▶ **SLOPED ROOF SNOWLOAD:** Uniform load on horizontal projection of a sloped roof, also known as the balanced load.



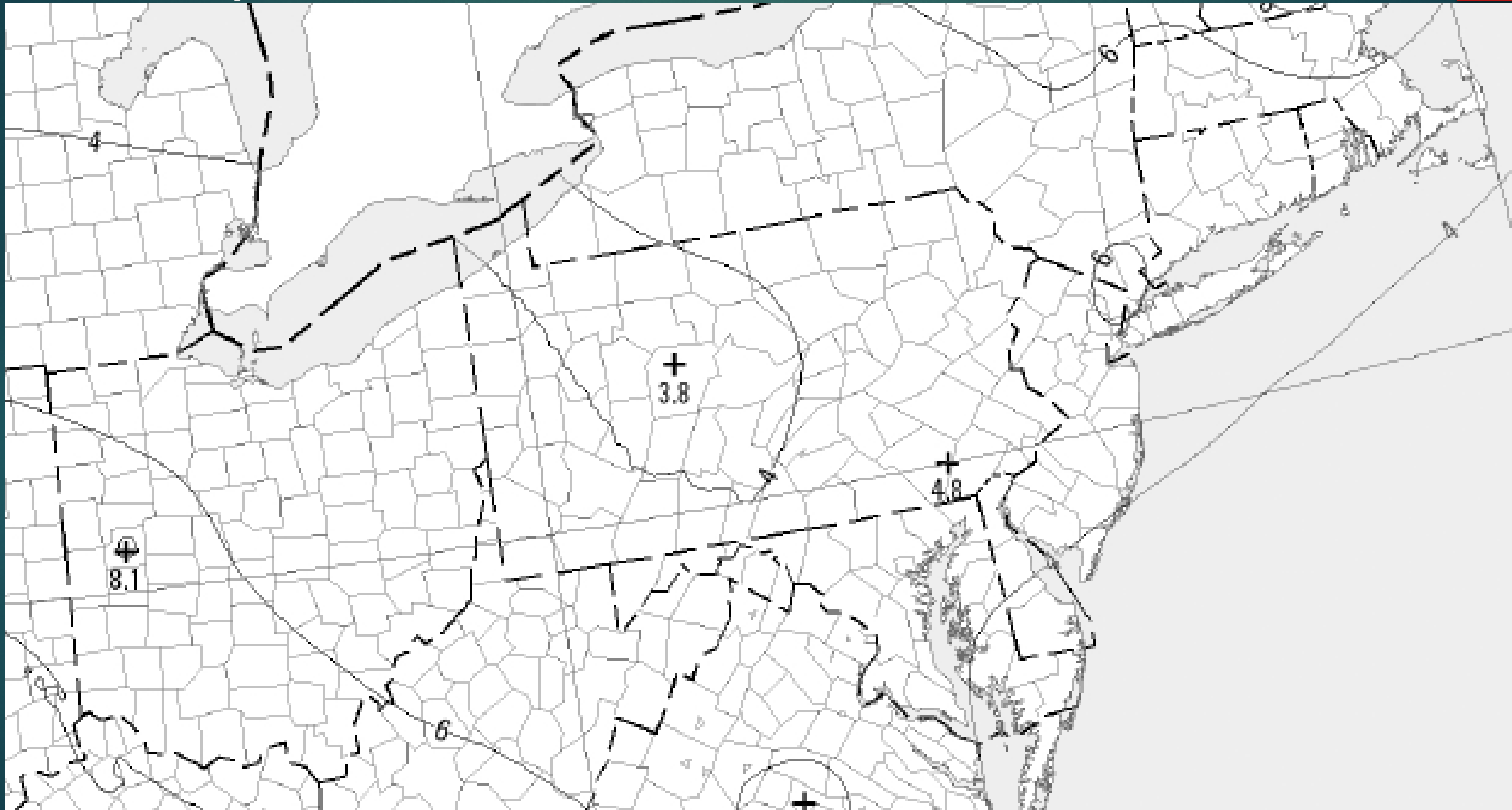
Wind Loads



Earthquake Loads



Earthquake Loads



Soil Loads

**TABLE 1610.1
LATERAL SOIL LOAD**

| DESCRIPTION OF BACKFILL MATERIAL ^c | UNIFIED SOIL CLASSIFICATION | DESIGN LATERAL SOIL LOAD ^a (pound per square foot per foot of depth) | |
|---|-----------------------------|--|------------------|
| | | Active pressure | At-rest pressure |
| Well-graded, clean gravels; gravel-sand mixes | GW | 30 | 60 |
| Poorly graded clean gravels; gravel-sand mixes | GP | 30 | 60 |
| Silty gravels, poorly graded gravel-sand mixes | GM | 40 | 60 |
| Clayey gravels, poorly graded gravel-and-clay mixes | GC | 45 | 60 |
| Well-graded, clean sands; gravelly sand mixes | SW | 30 | 60 |
| Poorly graded clean sands; sand-gravel mixes | SP | 30 | 60 |
| Silty sands, poorly graded sand-silt mixes | SM | 45 | 60 |
| Sand-silt clay mix with plastic fines | SM-SC | 45 | 100 |
| Clayey sands, poorly graded sand-clay mixes | SC | 60 | 100 |
| Inorganic silts and clayey silts | ML | 45 | 100 |
| Mixture of inorganic silt and clay | ML-CL | 60 | 100 |
| Inorganic clays of low to medium plasticity | CL | 60 | 100 |
| Organic silts and silt clays, low plasticity | OL | Note b | Note b |
| Inorganic clayey silts, elastic silts | MH | Note b | Note b |
| Inorganic clays of high plasticity | CH | Note b | Note b |
| Organic clays and silty clays | OH | Note b | Note b |



Load Combinations

Where allowable stress design (working stress design), as permitted by this code, is used, structures and portions thereof shall resist the most critical effects resulting from the following combinations of loads:

- ▶ $D + F$ (Equation 16-8)
- ▶ $D + H + F + L$ (Equation 16-9)
- ▶ $D + H + F + (L_r \text{ or } S \text{ or } R)$ (Equation 16-10)
- ▶ $D + H + F + 0.75(L) + 0.75(L_r \text{ or } S \text{ or } R)$ (Equation 16-11)
- ▶ $D + H + F + (0.6W \text{ or } 0.7E)$ (Equation 16-12)
- ▶ $D + H + F + 0.75(0.6W) + 0.75L + 0.75(L_r \text{ or } S \text{ or } R)$ (Equation 16-13)
- ▶ $D + H + F + 0.75(0.7E) + 0.75L + 0.75S$ (Equation 16-14)
- ▶ $0.6D + 0.6W + H$ (Equation 16-15)
- ▶ $0.6(D + F) + 0.7E + H$ (Equation 16-16)



Stress (rectangular cross-section)

▶ Axial - $f_a = \frac{P}{A} = \frac{P}{bd}$

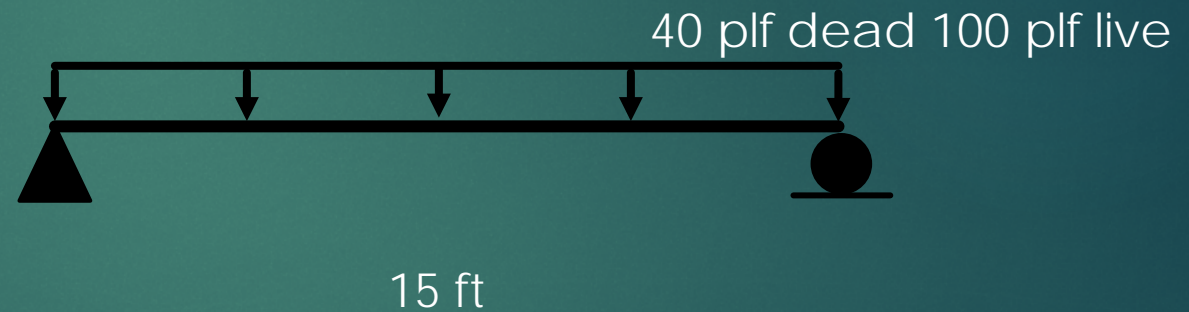
▶ Flexural - $f_b = \frac{M}{S} = \frac{M}{b} \frac{6}{d^2}$

▶ Shear - $f_v = \frac{3V}{2A} = \frac{3V}{2bd}$



Example

- ▶ 2x12 No2 DF-L 15 ft long simple beam with 40 plf dead load and 100 plf live load



$$V_{max} = \frac{wl}{2} = \frac{(140plf)(15ft)}{2} = 1050lbs$$

$$M_{max} = \frac{wl^2}{8} = \frac{(140plf)(15ft)^2}{8} = 3937.5ft - lbs$$

$$\delta_{max} = \frac{5wl^4}{384EI} = \frac{5(100plf)(15ft)^4 \left(\frac{12in}{ft}\right)^3}{384(1.6E6psi)(178in^4)} = 0.4in$$

$$\frac{l}{360} = \frac{(15ft) \left(\frac{12in}{ft}\right)}{360} = 0.5in$$



$$f_b = \frac{(3937.5 \text{ ft} - \text{lbs}) 6 \left(\frac{12 \text{ in}}{\text{ft}} \right)}{(1.5 \text{ in})(11.25 \text{ in})^2} = 1493 \text{ psi}$$

$$f_v = \frac{3 (1050 \text{ lbs})}{2 (1.5 \text{ in})(11.25 \text{ in})} = 93.3 \text{ psi}$$



Questions

